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Quantum signal processing in electron quantum optics

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G. Fève, A. Marguerite, R. Bisognin (LPA, ENS Paris)

A. Marguerite *et al*, Physica Status Solidi B **254**, 1600618 (2017)

B. Roussel *et al*, Physica Status Solidi B **254**, 1600621 (2017)

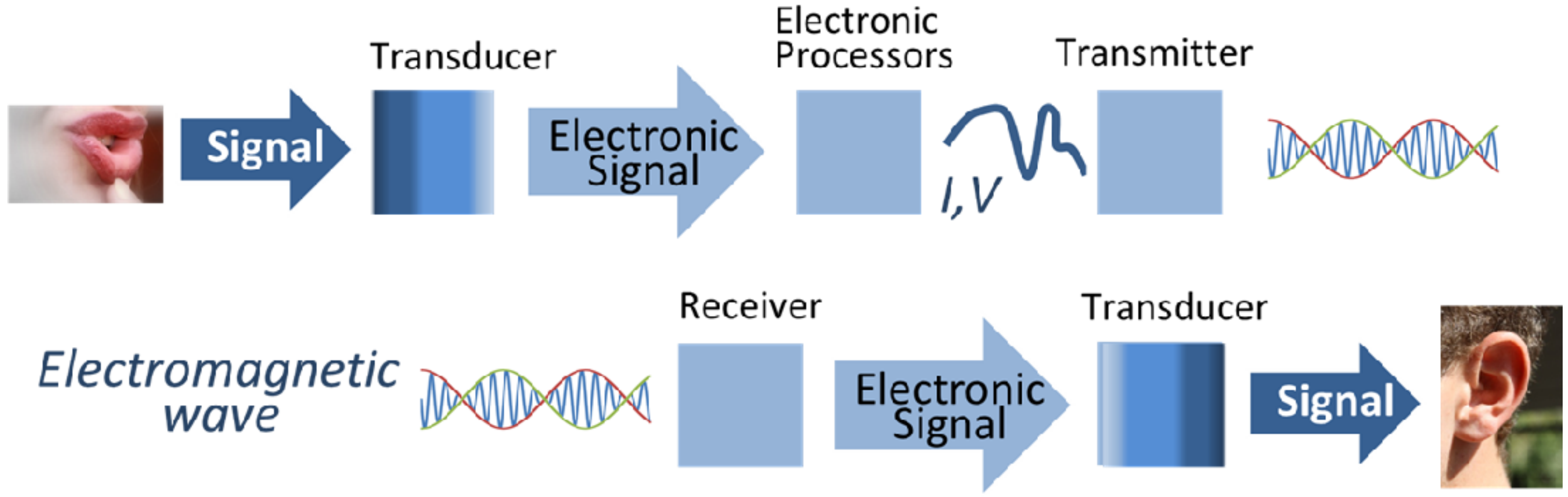
A. Marguerite *et al*, arXiv:1710.11181

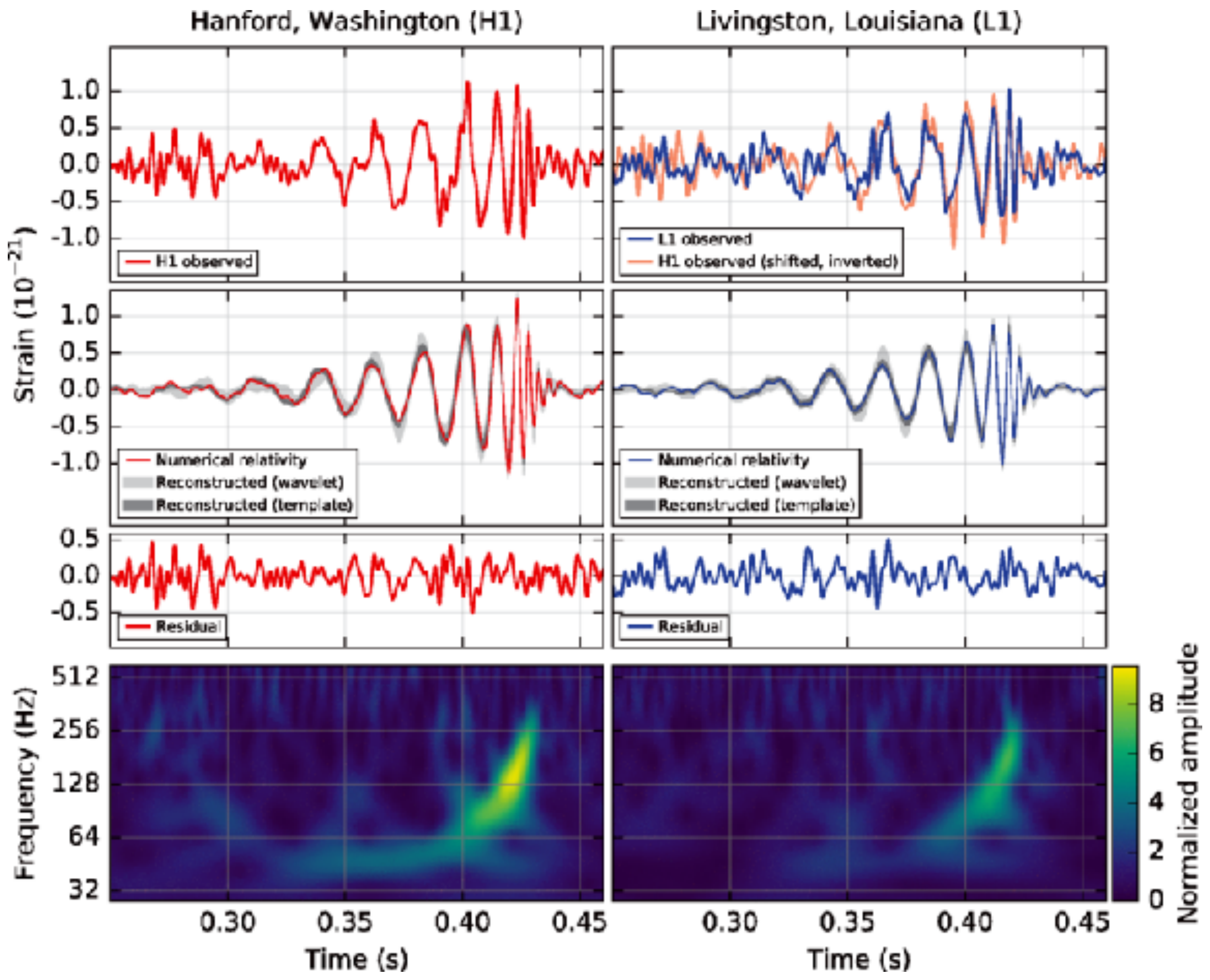
B. Roussel, PhD thesis (tel-01730943)

Plan

- Introduction
- Lessons from quantum optics
- Electron quantum optics
- Signal processing for quantum electrical currents
- Conclusion & perspectives

Signal processing

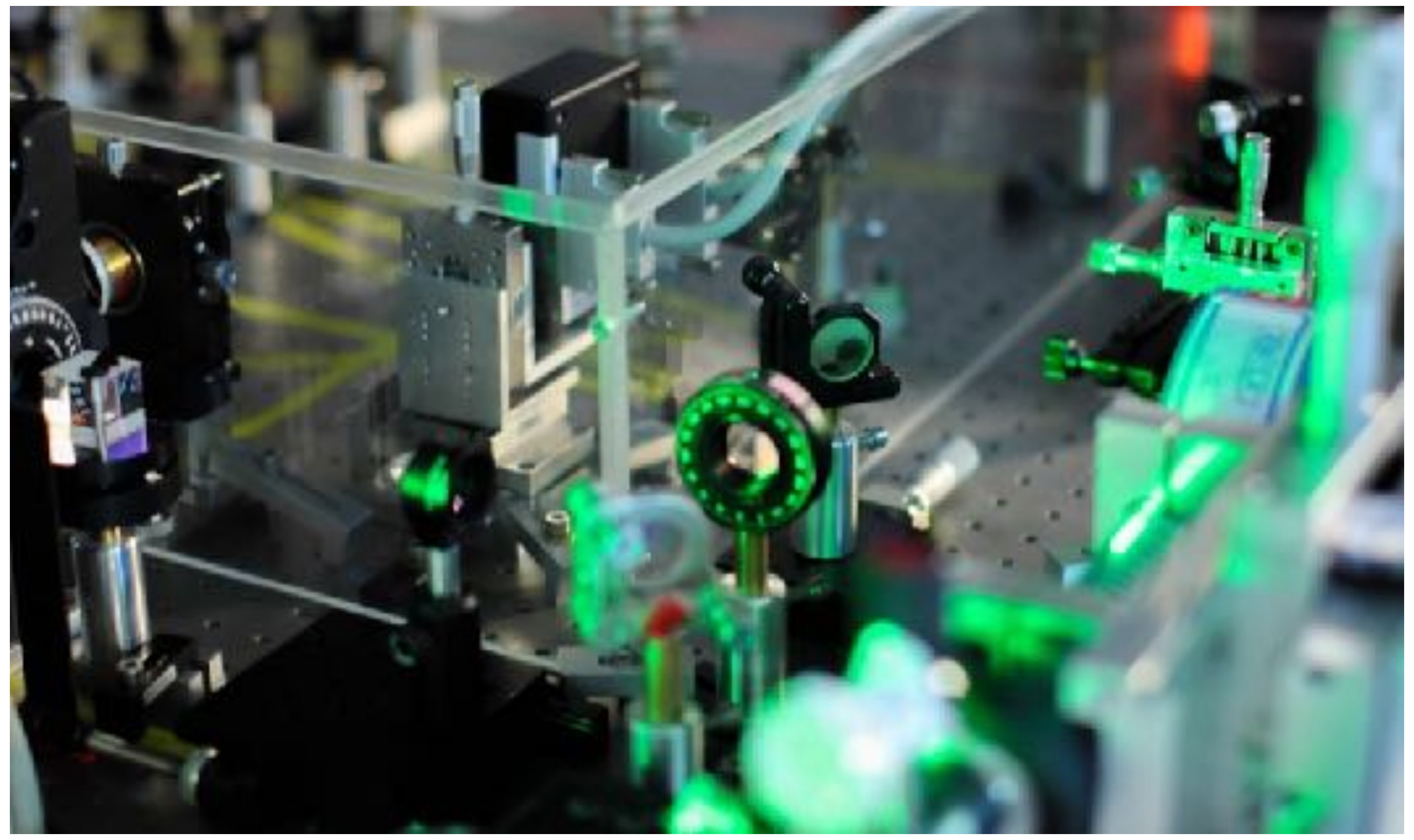




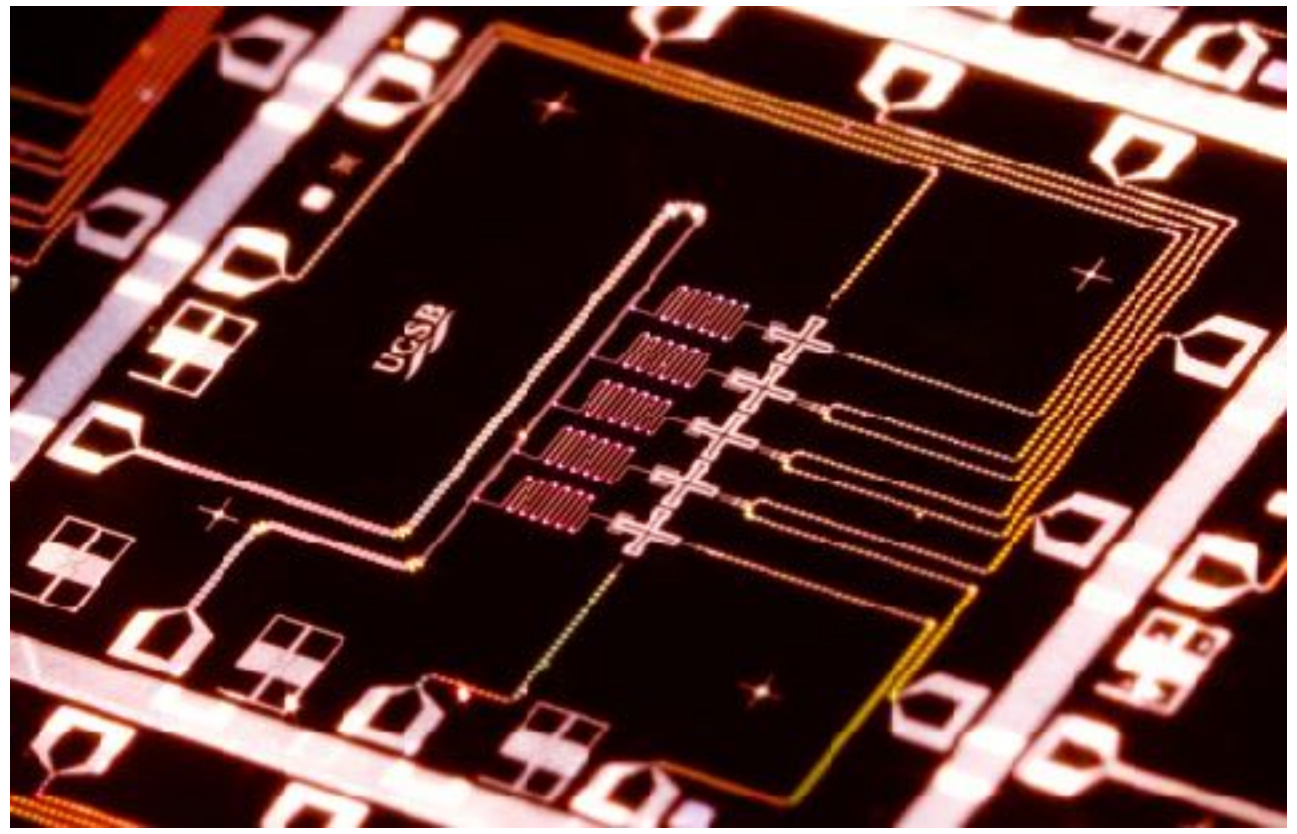
An enabling technology that aims at processing, transferring and retrieving information carried in various physical formats called « signals »…

J. Mourra, IEEESignal Process. Mag **26**, 6 (2009).

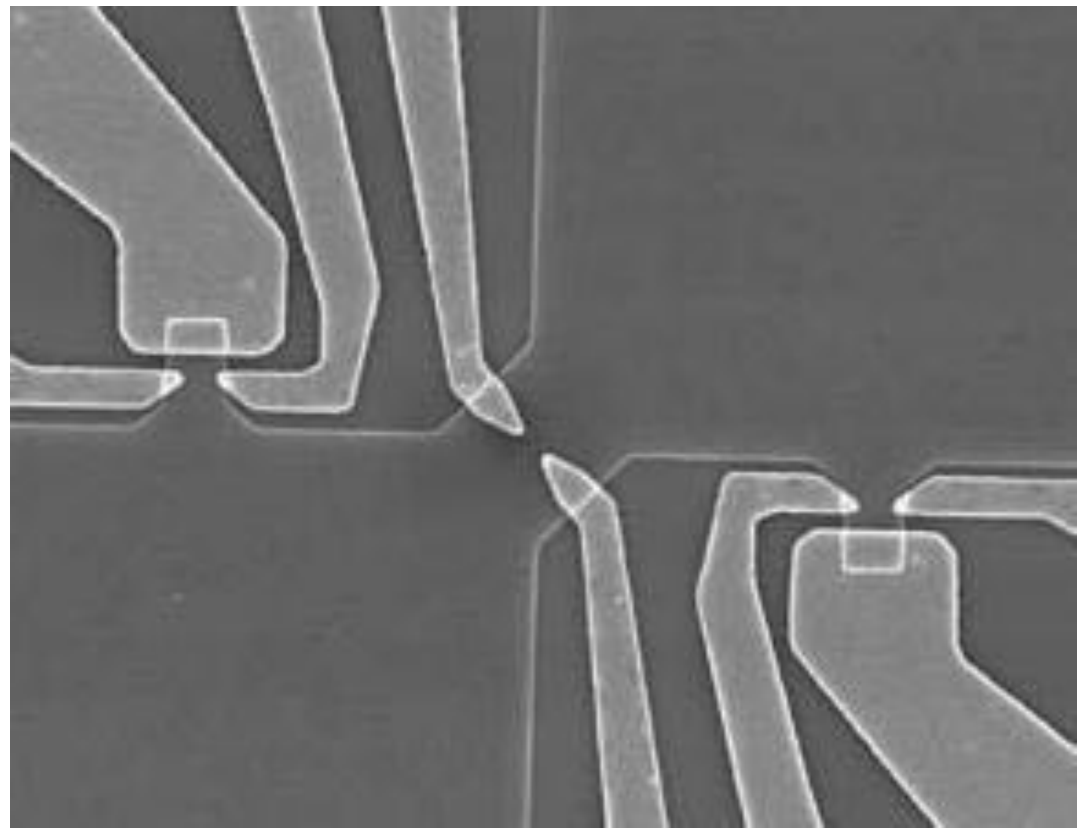
Quantum signal processing



Light beams



Microwave radiation



Electrical currents

An enabling technology that aims at processing, transferring and retrieving classical or quantum information carried by various quantum states called « quantum signals »…

Main questions

How to characterize the state of a quantum beam ?

- What are the quantum signals carried by the beam ?
- How to describe them in a simple way ?

Experimental aspects

- Sources ?
- Analyzers ?

Theoretical aspects

- Description of a quantum beam ?
- Tomography ?
- Signal processing ?

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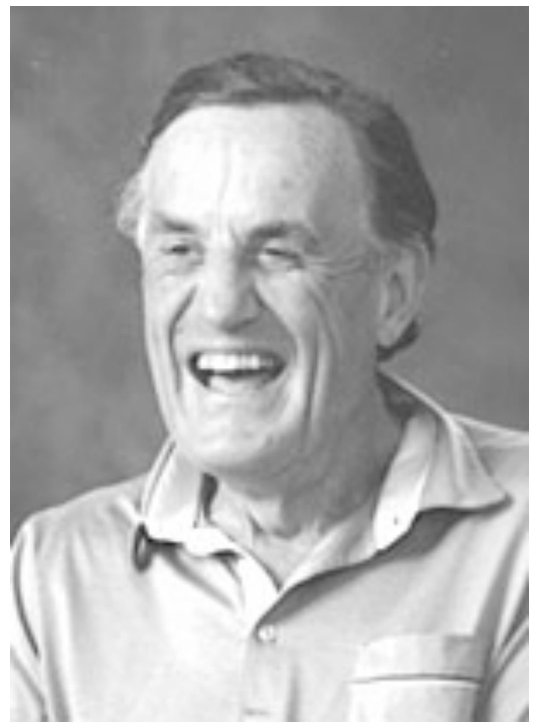
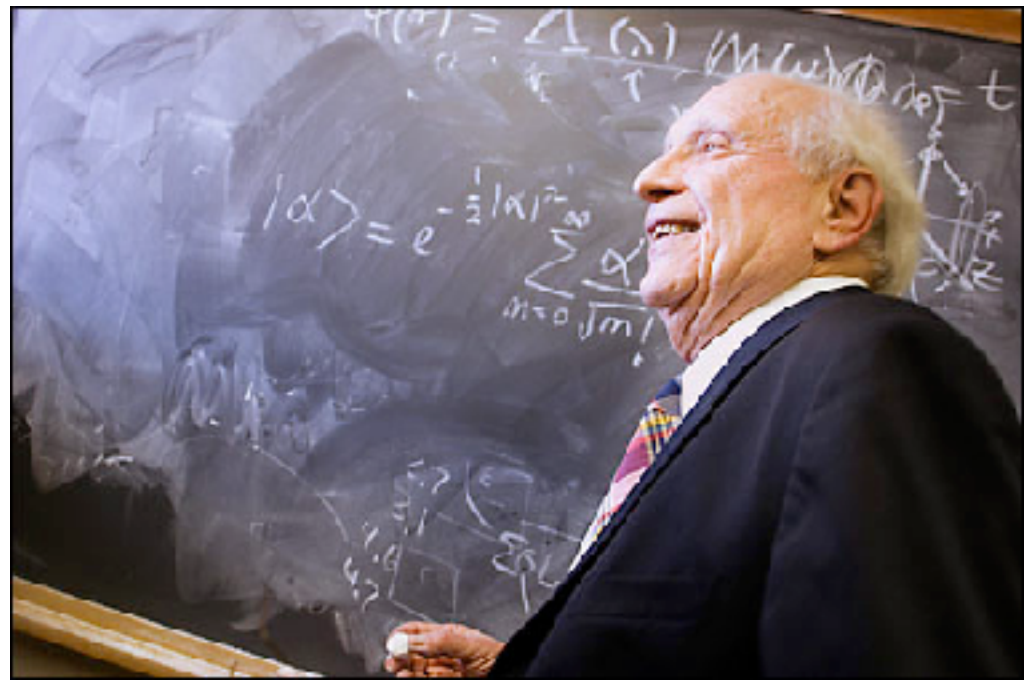
From classical to quantum optics

J.C. Maxwell



R. Hanbury Brown

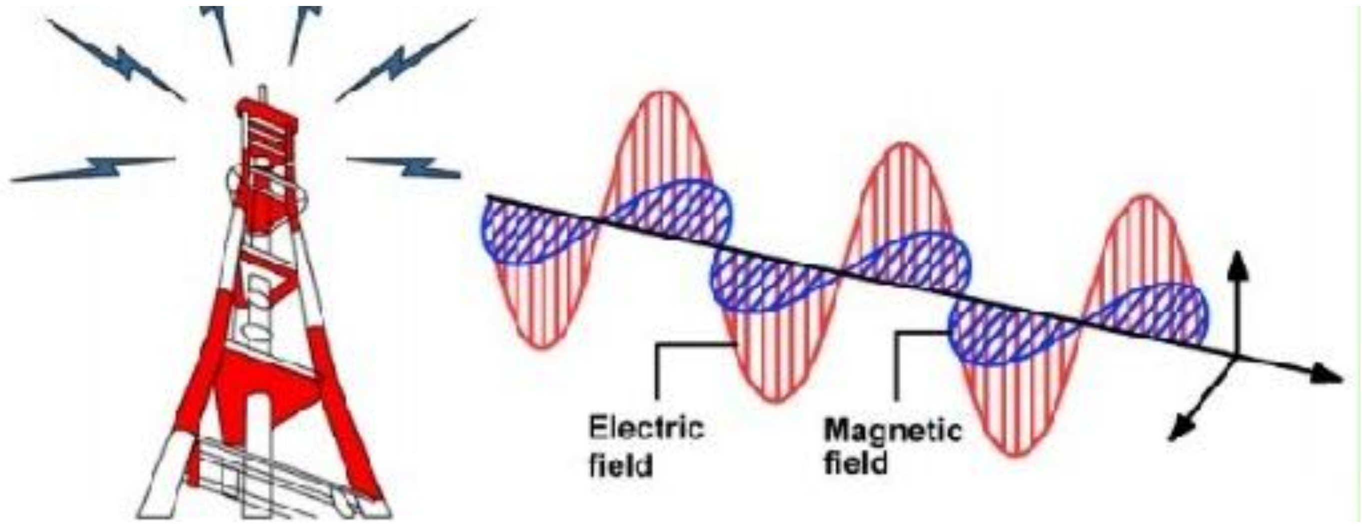


R.J. Glauber

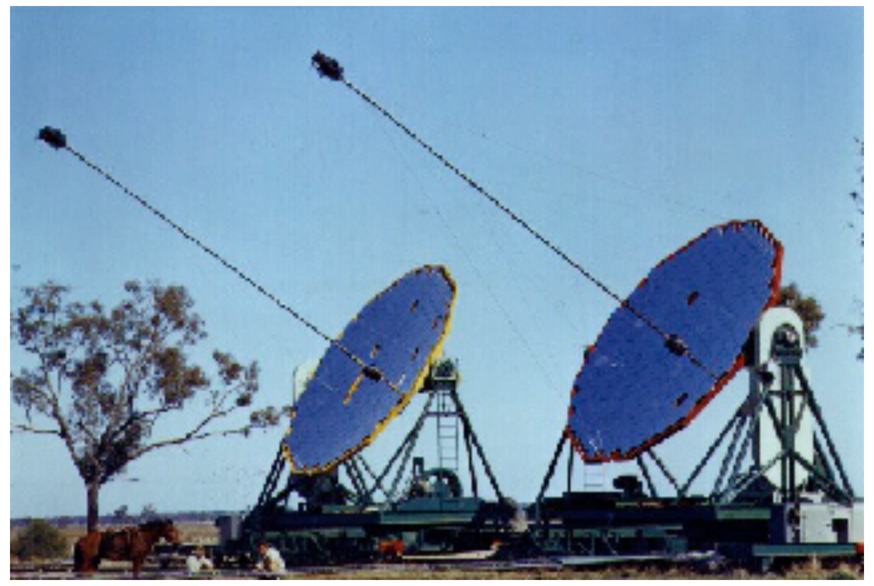
The Quantum Theory of Optical Coherence*

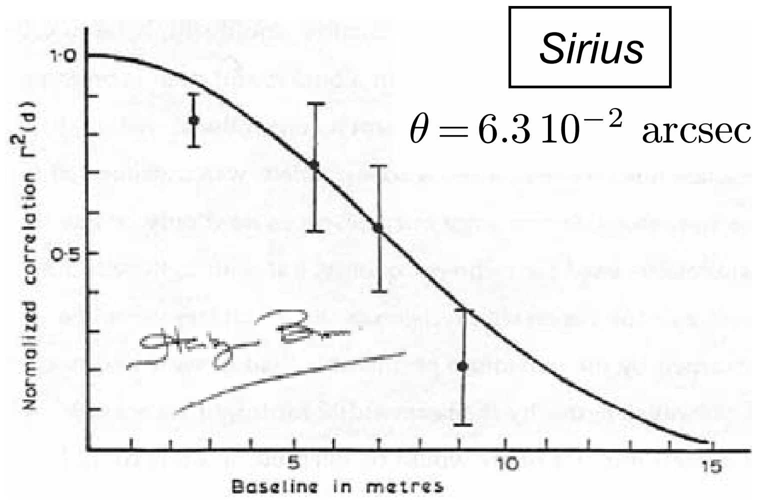
Roy J. Glauber
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received 11 February 1963)

Phys. Rev. **130**, 2529 (1963)
Phys. Rev. Lett. **10**, 84 (1963)
Phys. Rev. **131**, 2766 (1963)



1956: stellar interferometry...

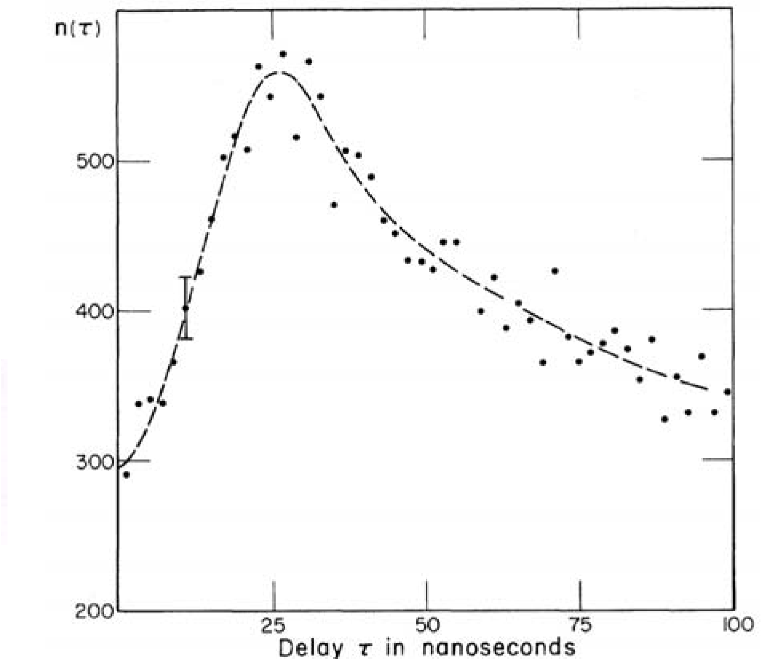




$\Gamma^2(d) = [2J_1(x)/x]^2$; $x = \pi\theta d/\lambda$

Nature **178**, 1046 (1956)

1977: non classical light
resonance light from a single atom

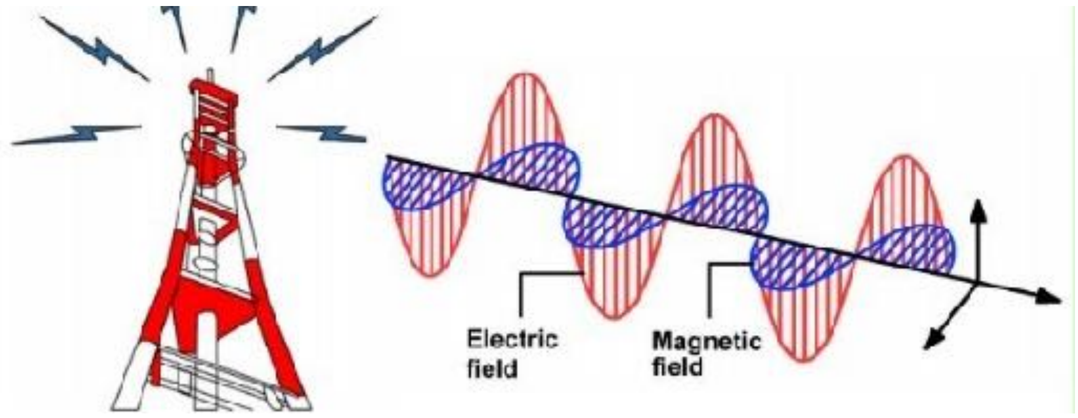


Phys. Rev. Lett. **39**, 691 (1977)

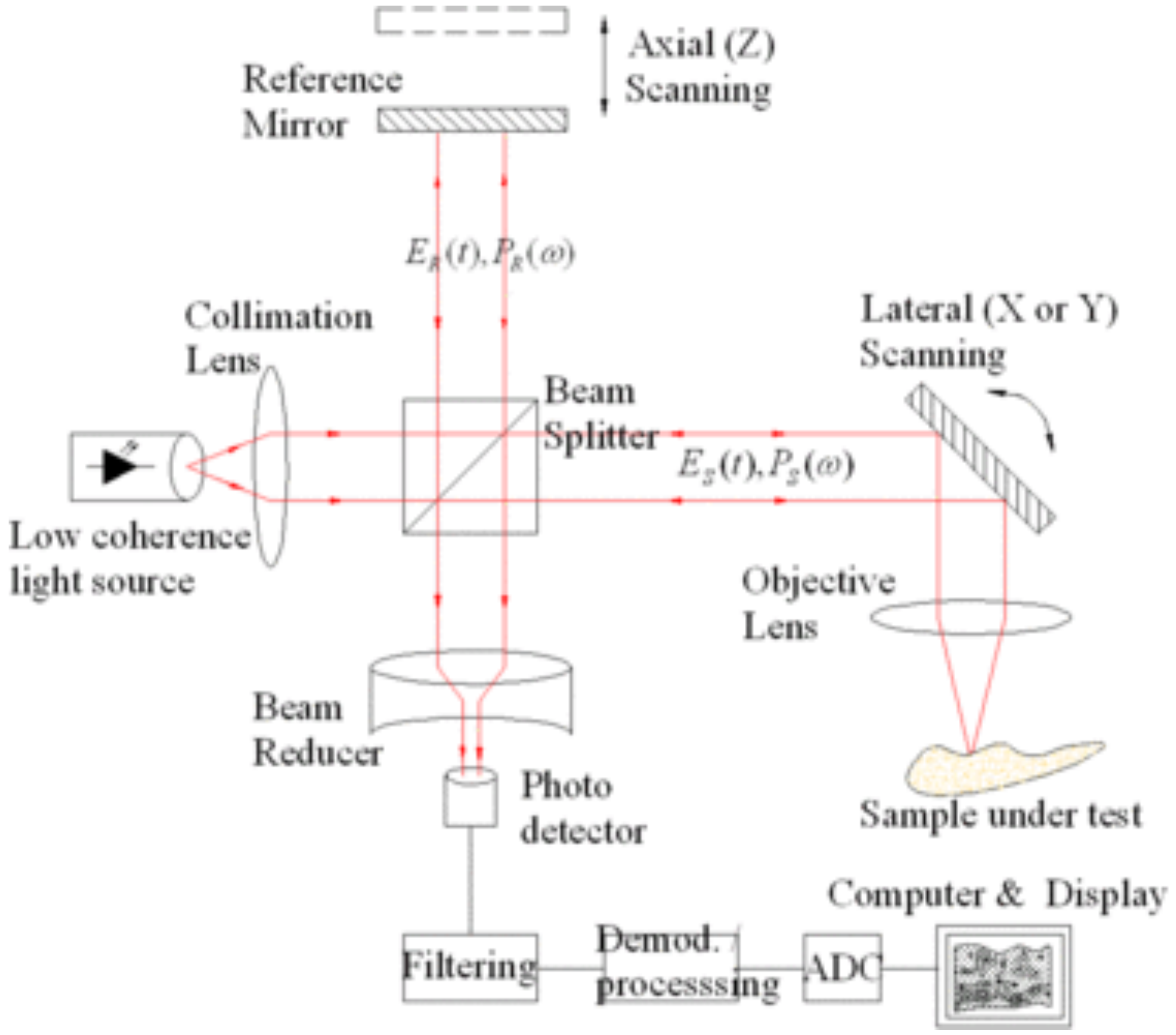
Classical waves

Classical field amplitude $\vec{\mathcal{E}}(\mathbf{r},t)$

- Maxwell's equations determine the fields from the currents
- Laplace's force determines charge evolution from fields

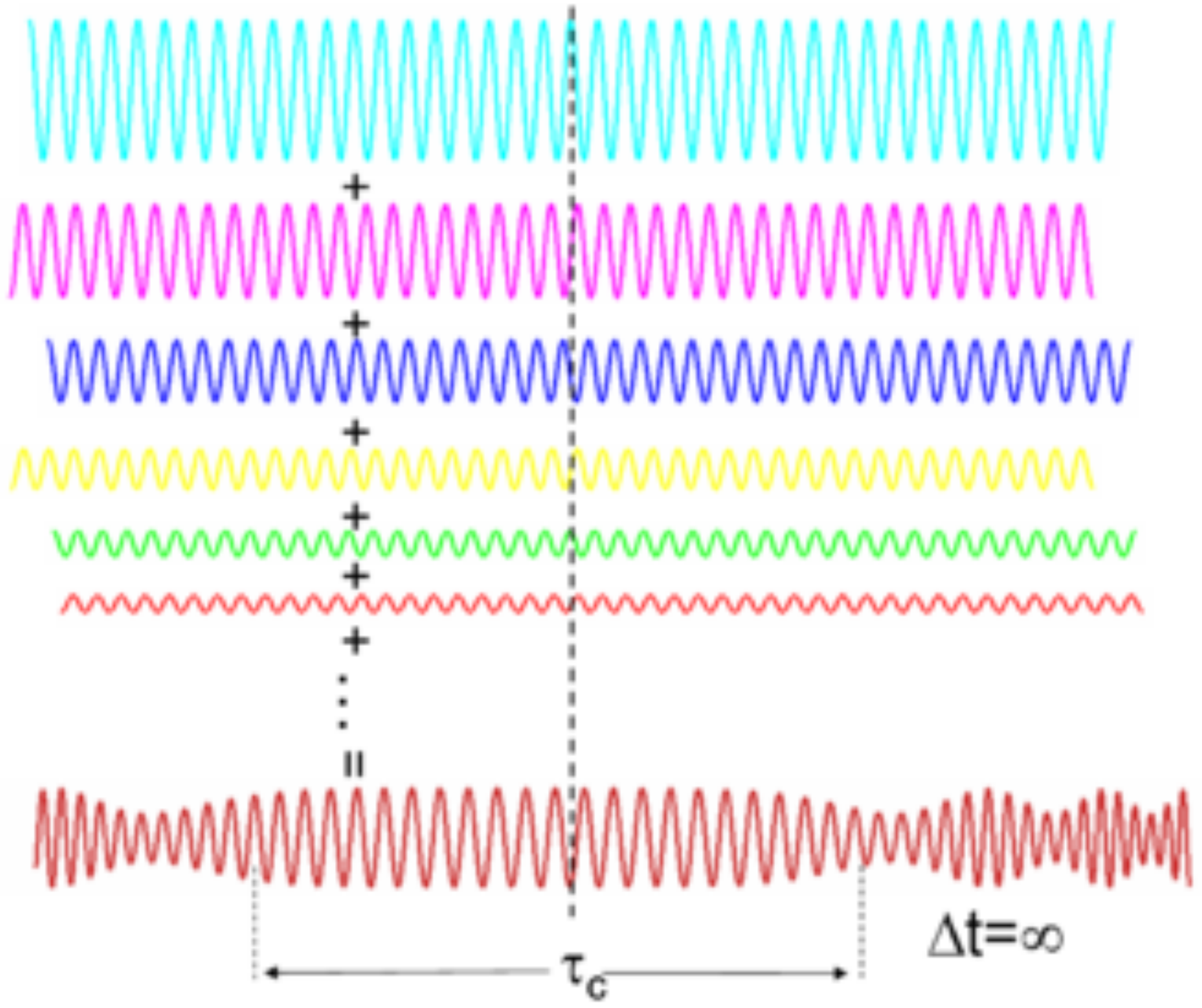


Classical fluctuations of the field $\mathbb{E}\left(\vec{\mathcal{E}}(\mathbf{r},t)\vec{\mathcal{E}}(\mathbf{r}',t')\right)$



- Theory of optical coherence: determines contrast in interferometers

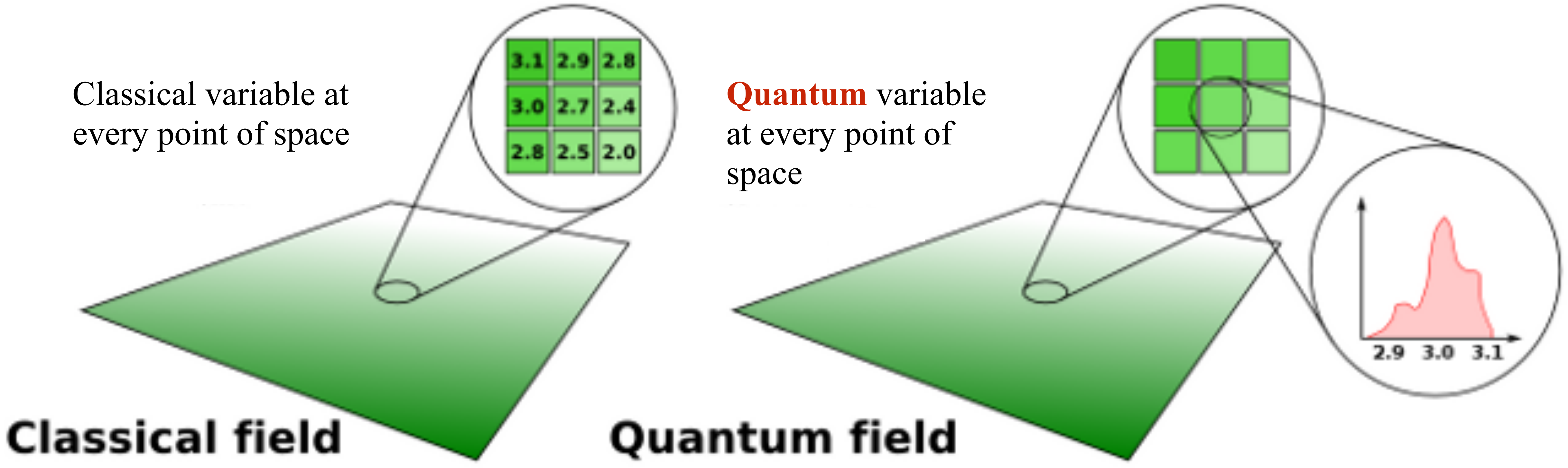
- Optical coherence interferometry: reconstructing images by exploiting interferometry with low coherence light.



Quantum beams of light

Quantum electrodynamics

- Electromagnetic fields becomes quantum
- Built-in light matter coupling

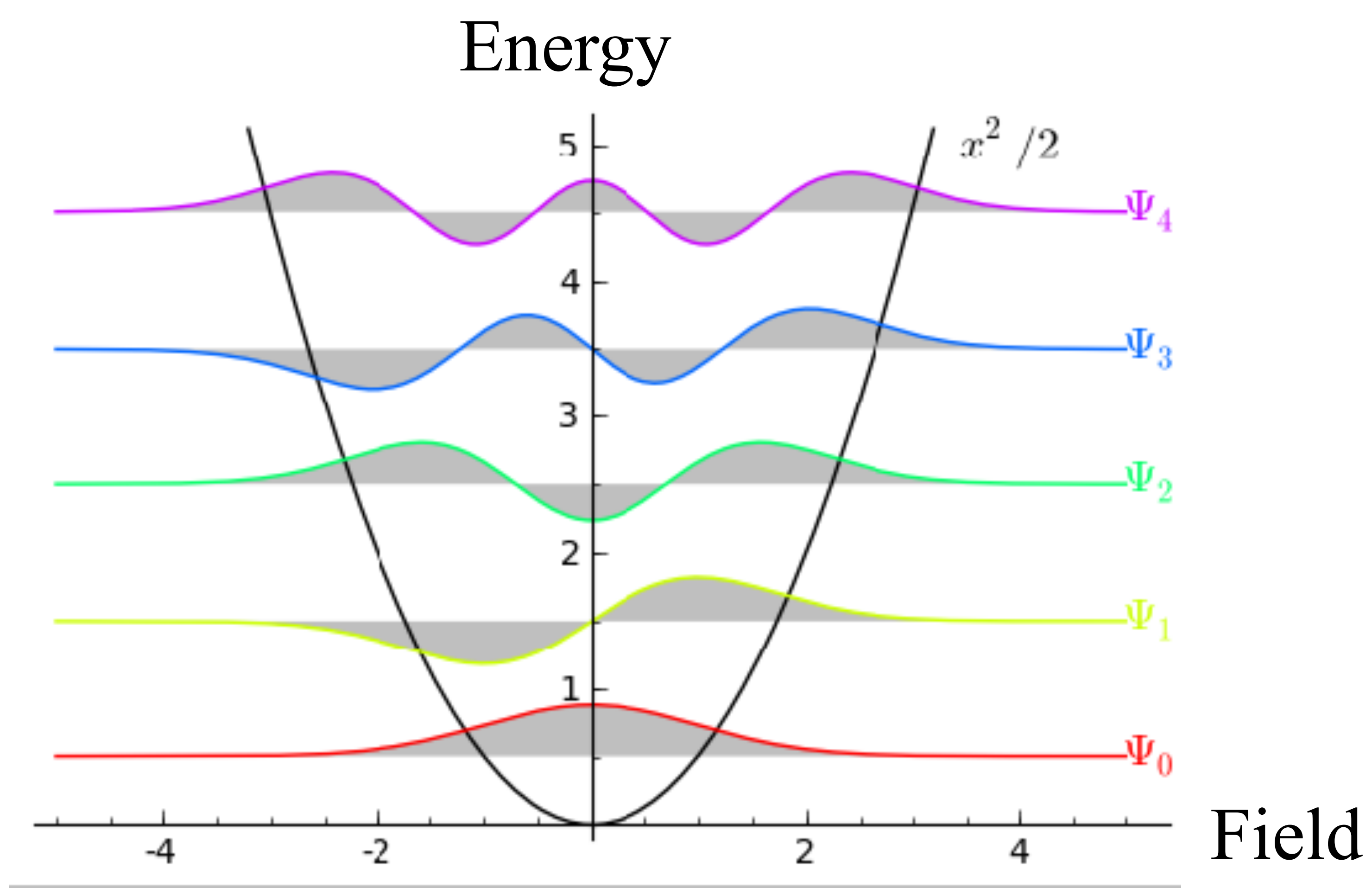


What are photons ?

- Excitations on top of the vacuum
- Carry energy and momentum (particle attributes)

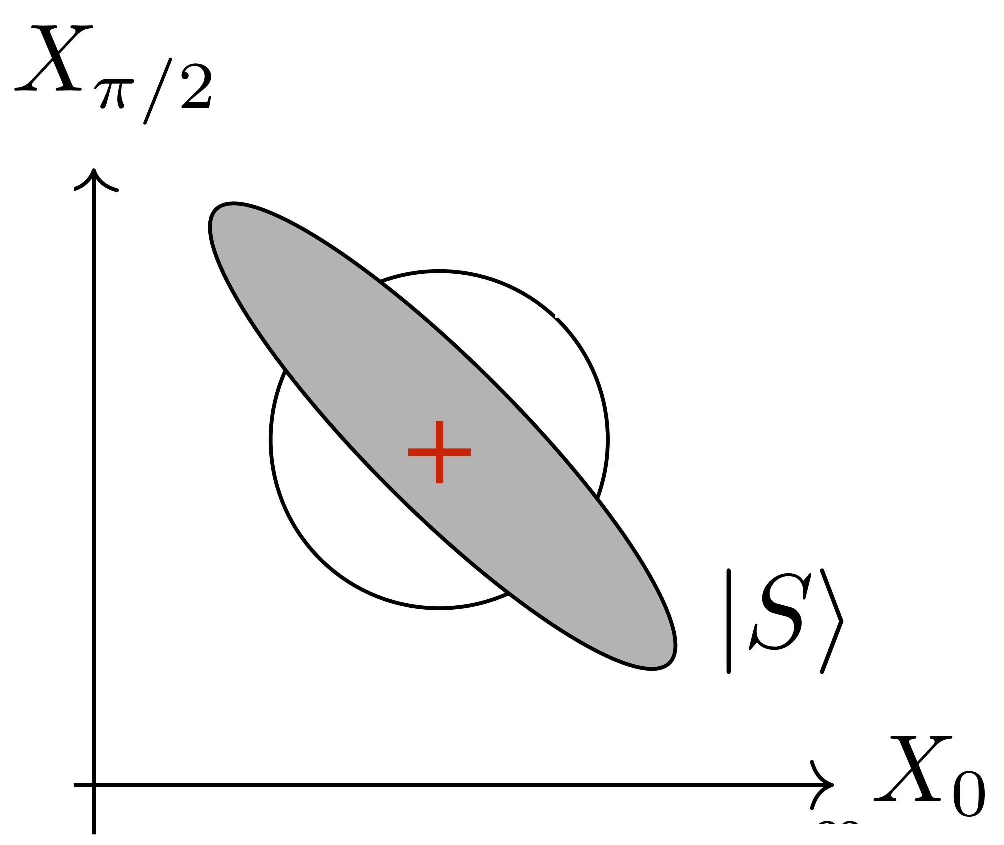
$$\mathbf{E}(\mathbf{r},t) = \underbrace{\mathbf{E}^{(+)}(\mathbf{r},t)}_{\text{photon destruction}} + \underbrace{\mathbf{E}^{(-)}(\mathbf{r},t)}_{\text{photon creation}}$$

photon destruction (positive frequencies) — photon creation (negative frequencies)



Fixed frequency modes

Experimentally accessed quantities



Field amplitudes $\quad \vec{\mathcal{E}}(\mathbf{r},t) = \langle \mathbf{E}(\mathbf{r},t)\rangle_\rho$

Quantum fluctuations

$$X_\theta = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\theta} a + \mathrm{e}^{-\mathrm{i}\theta} a^\dagger\right)$$

$$\langle (\Delta X_\theta)^2\rangle_\rho = \frac{1}{2}\langle a\, a^\dagger + a^\dagger a\rangle_\rho - |\langle a\rangle_\rho|^2 + \Re\left(\mathrm{e}^{2i\theta}(\langle a^2\rangle_\rho - \langle a\rangle_\rho^2)\right)$$

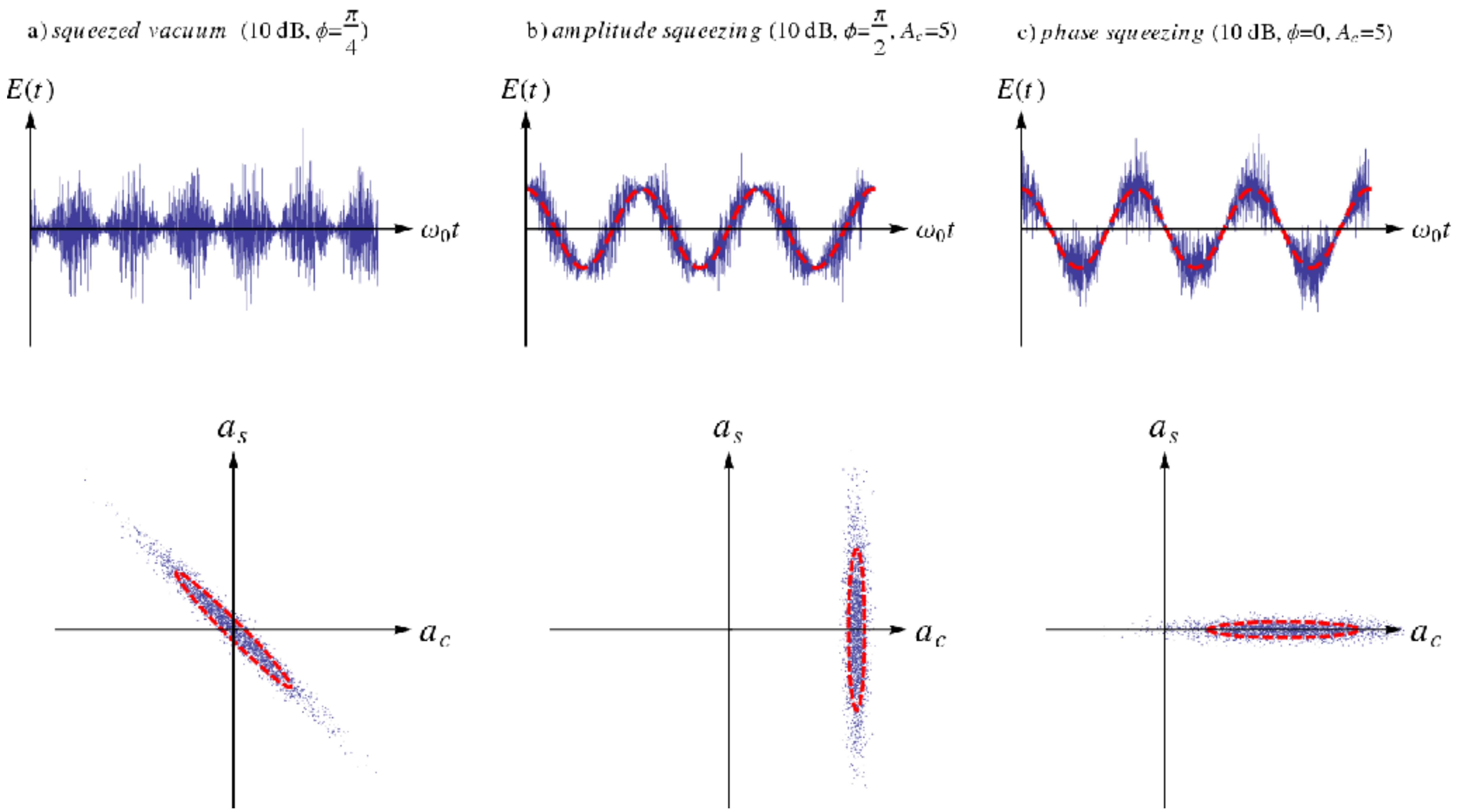
$$G^{(1)}(\mathbf{r},t|\mathbf{r}',t') = \mathrm{Tr}\left(\mathbf{E}^{(+)}(\mathbf{r},t)\,\rho\,\mathbf{E}^{(-)}(\mathbf{r}',t')\right)$$

First order coherence

$$\mathrm{Tr}\left(\mathbf{E}^{(+)}(\mathbf{r}',\mathbf{t}')\mathbf{E}^{(+)}(\mathbf{r},\mathbf{t})\,\rho\right)$$

Pair amplitude

Up to second moments



Zero average field
Time dependent fluctuations

Non zero average field
Time dependent fluctuations

Pros and cons

- Only requires up to noise measurements (electronics)
- OK for Gaussian fluctuations but not generically enough
- Non classical states: squeezing

Applications

- Quantum sensing for interferometers (LIGO)
- Enhanced precision quantum imaging Phys. Rev. Lett. **88**, 203601 (2002)
- Multimode entanglement: quantum communication, quantum imaging (Photonics **76**, 32-35, (2015))

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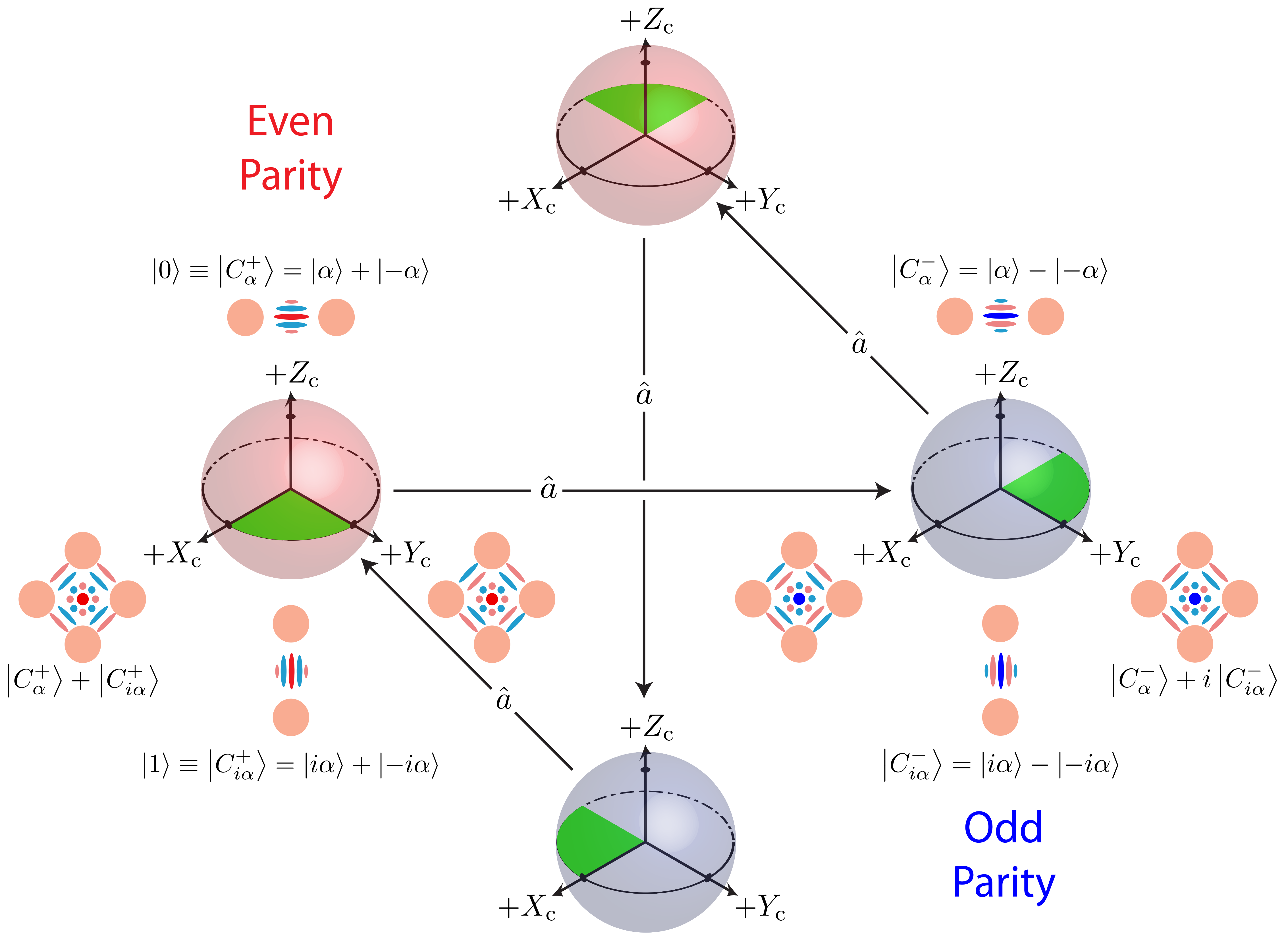
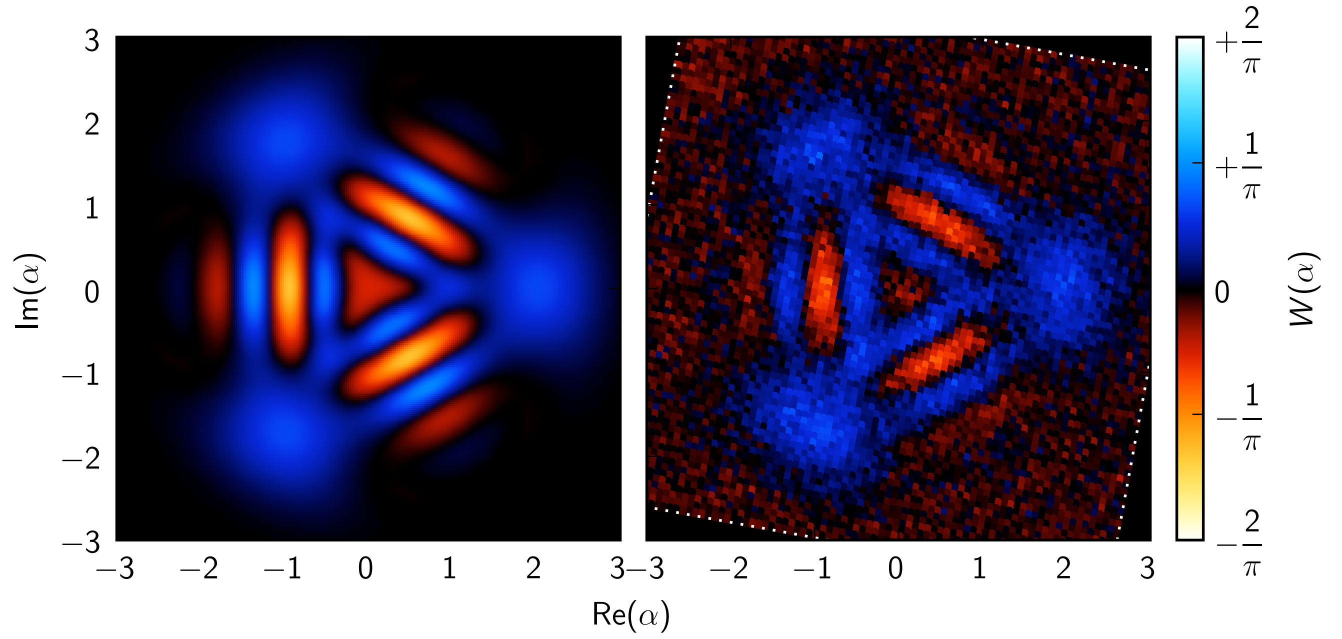
Full tomography



Pros and cons

- Requires the full access to statistics & Max Like, Max Ent methods…
- Visualization using the Wigner distribution function in Fresnel plane
- Not yet fully multimode!



Applications

- Decoherence studies (Nature. **455**, 510 (2008))
- CV computation using Gottesman, Kitaev & Preskill logical qubit (Phys. Rev. **A** 64, 012310 (2001))
- “Cat code” encoding of qubits in non classical superpositions (Nature **536**, 441 (2016)).

Take home message #1

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What are the "(quantum) signals" carried by electromagnetic radiation ?

Classical signal

$$\overrightarrow{\mathcal{E}}(\mathbf{r}, t) = \langle \mathbf{E}(\mathbf{r}, t) \rangle_\rho$$

Statistical properties: classical coherence theory

Quantum signals

$$G^{(1)}(\mathbf{r}, t | \mathbf{r}', t') = \mathrm{Tr}\left(\mathbf{E}^{(+)}(\mathbf{r}, t)\, \rho\, \mathbf{E}^{(-)}(\mathbf{r}', t') \right)$$

$$\mathrm{Tr}\left(\mathbf{E}^{(+)}(\mathbf{r}', \mathbf{t}') \mathbf{E}^{(+)}(\mathbf{r}, \mathbf{t})\, \rho \right)$$

Quantum fluctuations: higher order coherence, photon statistics…

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Quantum optics : the art of controlling and processing quantum light signals

- Controlled generation of coherent excitations
- Measurement of their quantum coherence
- Quantum state reconstruction (*i.e.* quantum tomography)

How can you achieve this for electrical currents ?

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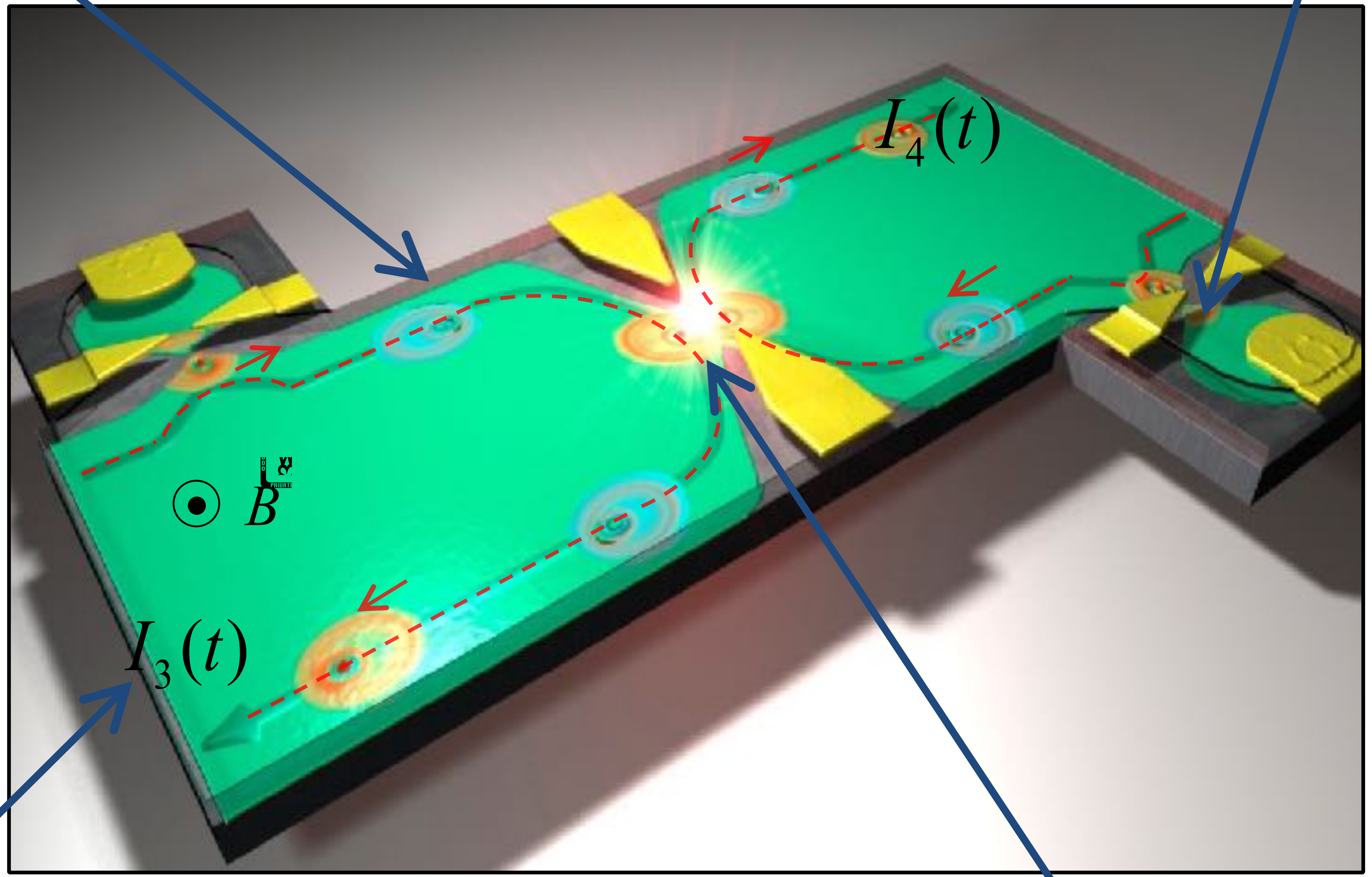
Electron quantum optics

Guided propagation
along 1D chiral edge channels

Single electron emitter



Measurement of output
current correlations

Quantum point contact used as electronic beam-splitter

Quantum Hall edge channels as electronic optical fibers

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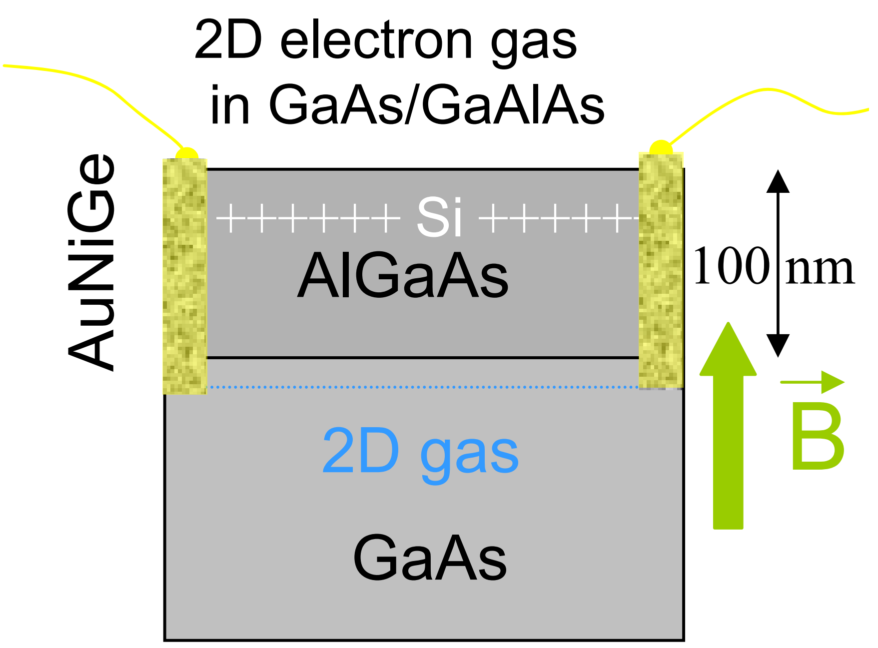
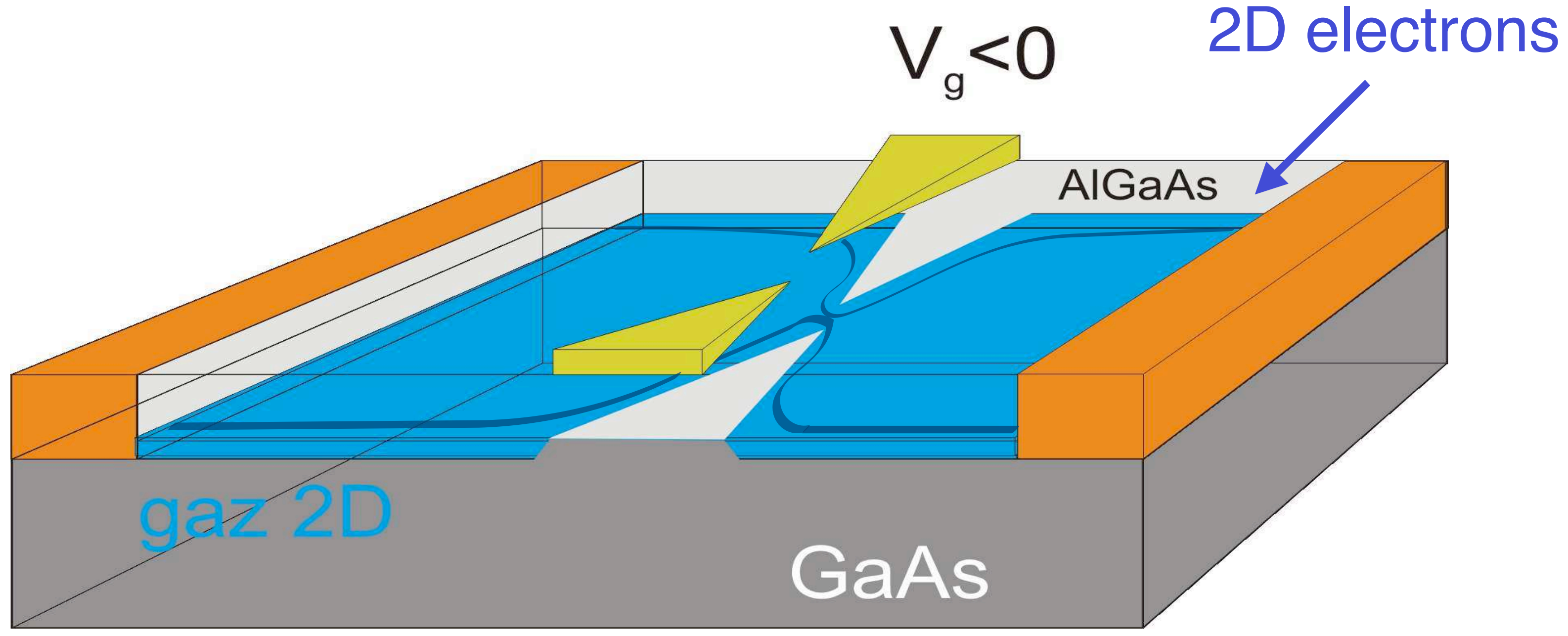
2DEG



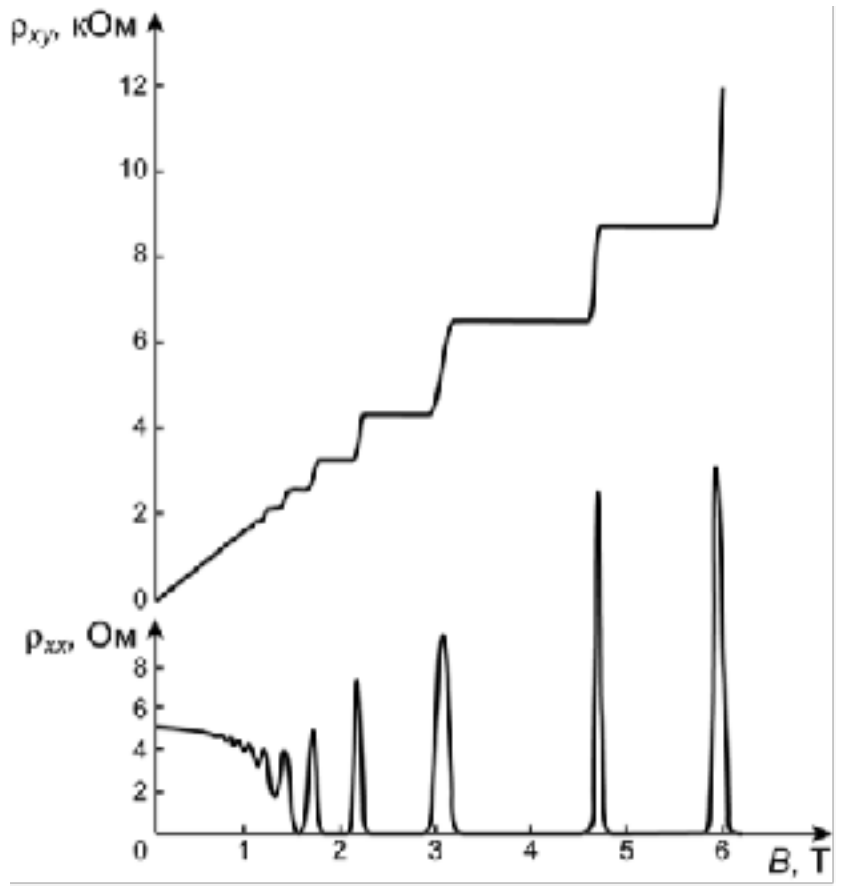


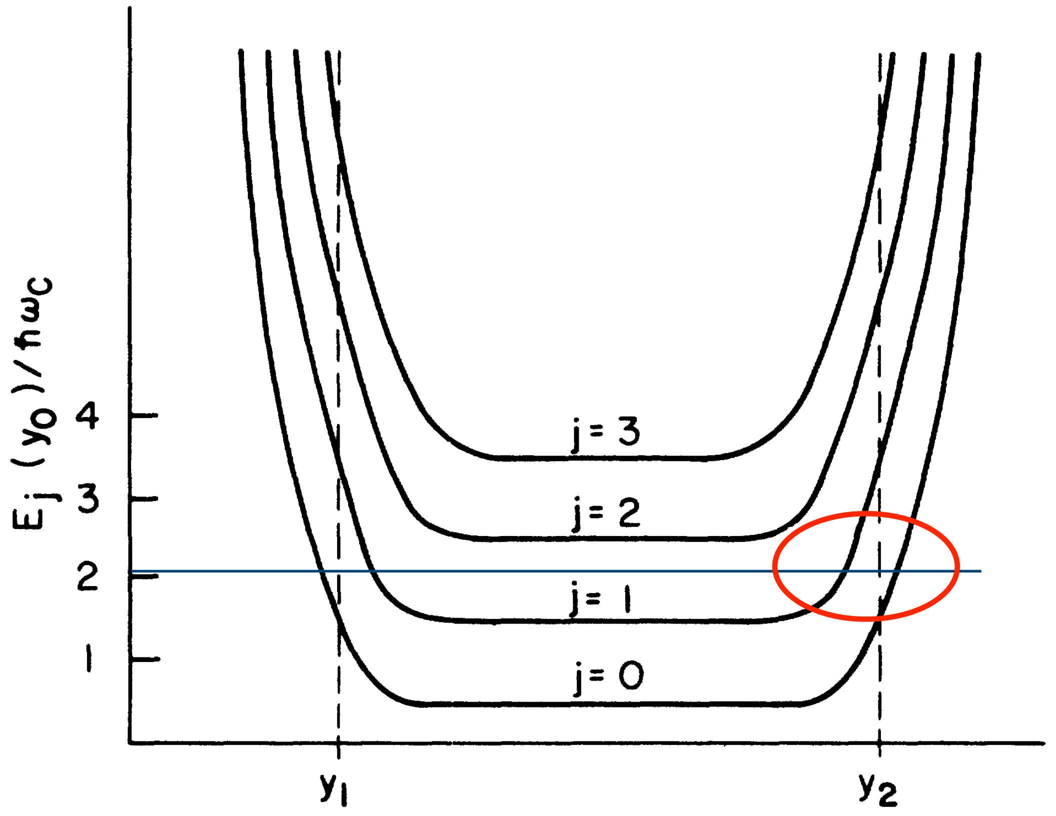
III-V semi-conductor heterojunction GaAs/GaAlAs

$$n \simeq 10^{11}\ \mathrm{cm}^{-2}$$

$$\mu \simeq 10^{6}\ \mathrm{cm}^2/\mathrm{VS}$$

Quantum Hall effect & edge channels





Insulating 2D bulk

Current transported along edge channels: **no backscattering!**

Chiral relativistic fermions

$$v_F \simeq 10^5 - 10^6\ \mathrm{m\,s}^{-1}$$

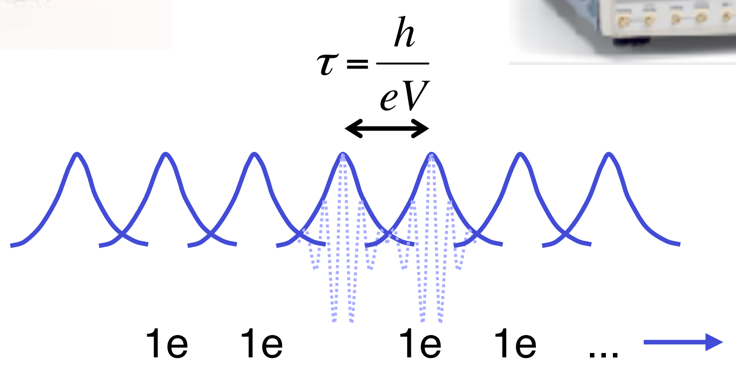
M. Büttiker, Phys. Rev. B. **88**, 9375 (1988)

New generators: single electron sources

Coherent nano-electronics:
many electrons sources

dc source

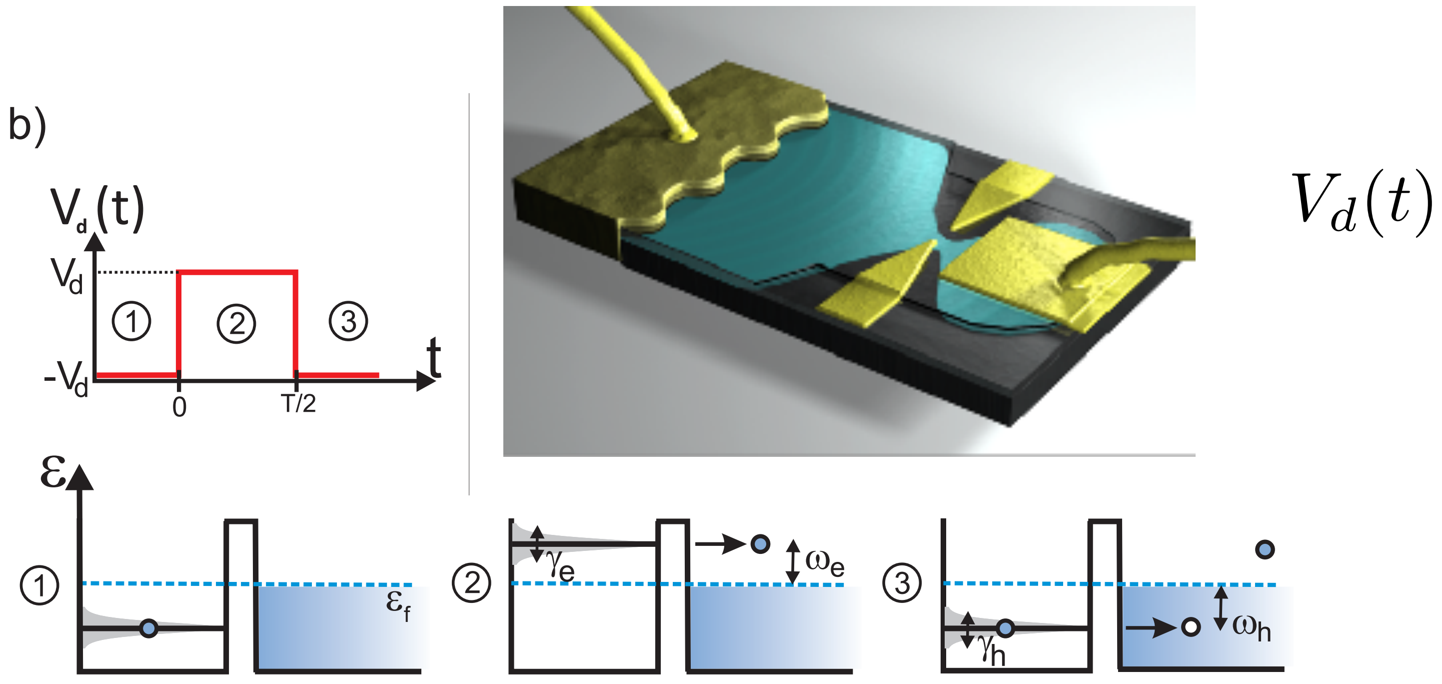
ac source



Many overlapping electrons!

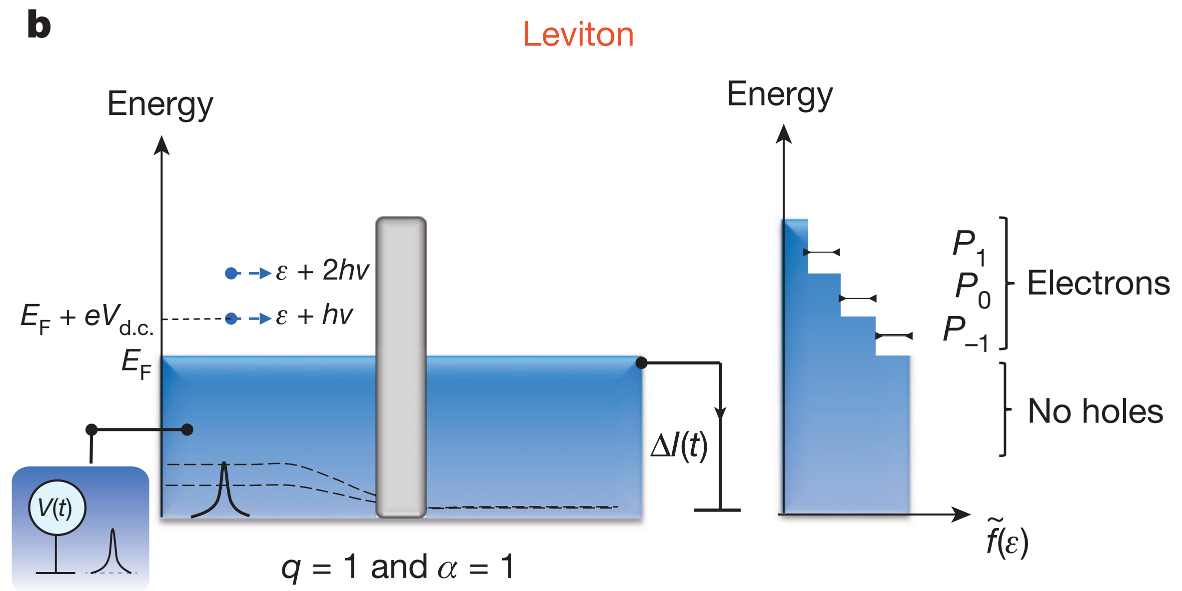
Electron quantum optics:
single or few electrons sources

Landau excitation source:



G. Fève *et al*, Science **316**, 1169 (2007)

Leviton source:



J. Dubois *et al*, Nature **502**, 659 (2013)

The basic questions of electron quantum optics

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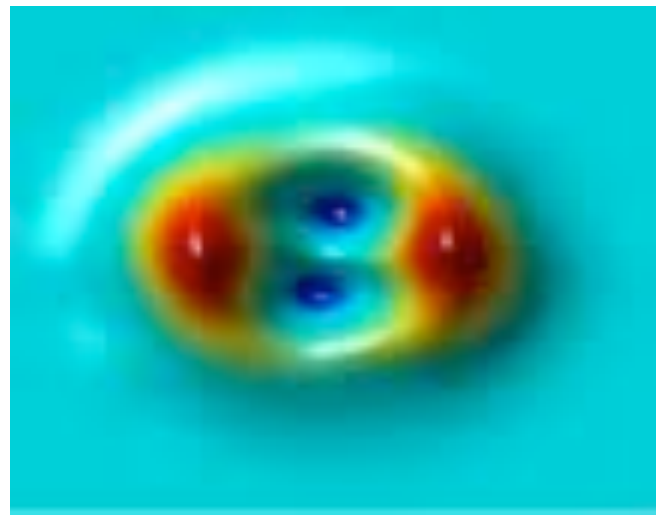
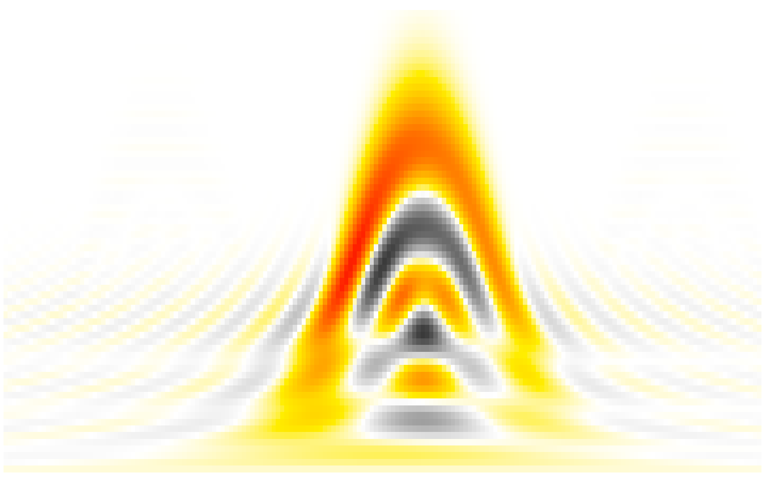
Photons	Electrons
classical wave limit — Bosons	Fermions — **no classical wave limit**
"True" vacuum	Fermi sea
Non interacting	Coulomb interactions
flying photons	**decoherence**
many quantas / single mode	**few quantas / many modes**
How is the mode occupied ?	Shape of emitted wave-packets ?





Electronic quantum signals

Quantum electronic sources and circuits emits quantum signals

Electron amplitude

$$\langle \psi(1) \rangle_\rho \qquad 1 = (x_1, t_1)$$

Parity superselection rule

$$\langle \psi(1) \rangle_\rho = 0$$

Electron pair amplitude

$$\langle \psi(1)\psi(2) \rangle_\rho$$

$\langle \psi(1)\psi(2) \rangle_\rho = 0$ Normal metal

$\langle \psi(1)\psi(2) \rangle_\rho \neq 0$ **Superconductivity**

B. Roussel, PhD thesis (tel-01730943, defended on Dec. 15th, 2017)

Quantum electronic sources and circuits emits quantum signals

Single electron coherence

$$\mathcal{G}_\rho^{(e)}(1|1') = \mathrm{Tr}(\psi(1)\rho\,\psi^\dagger(1'))$$

$$1 = (x_1, t_1)$$

Single particle properties:

- Average electrical current
- Average heat flow

Electronic decoherence

Two electron coherence

$$\mathcal{G}_\rho^{(2e)}(1,2|1',2') = \mathrm{Tr}(\psi(2)\psi(1)\rho\,\psi^\dagger(1')\psi(2'))$$

Two particle properties:

- Electrical current fluctuations
- Heat fluctuations

Correlations, entanglement

Review papers: E. Bocquillon *et al*, Ann. Phys. (Berlin) **526**, 1-30 (2014)
A. Marguerite *et al*, Physica Status Solidi B **254**, 1600618 (2017)

Single electron coherence: $\mathcal{G}^{(e)}_{\rho}(x,t|x',t') = \mathrm{Tr}(\psi(x,t)\,\rho\,\psi^{\dagger}(x',t'))$

Electronic analogue of Glauber's correlators $\mathcal{G}^{(1)}_{\rho}(x,t|x',t') = \mathrm{Tr}(E^{+}(x,t)\,\rho\,E^{-}(x',t'))$

Example: many body state $\prod_{k=1}^{N}\psi^{\dagger}[\varphi_k]\,|\emptyset\rangle$ with $\langle\varphi_k|\varphi_l\rangle = \delta_{k,l}$

$$\mathcal{G}^{(e)}(t|t') = \sum_{k=1}^{N}\varphi_k(-v_F t)\,\varphi_k(-v_F t')^{*}$$

Ideal single electron source: $\psi^{\dagger}[\varphi_e]|F\rangle$

Single electron coherence: $\mathcal{G}^{(e)}(t,t') = \mathcal{G}^{(e)}_{F}(t,t') + \varphi_e(-v_F t)\varphi_e(-v_F t')^{*}$

Fermi sea contribution — Wavepacket contribution

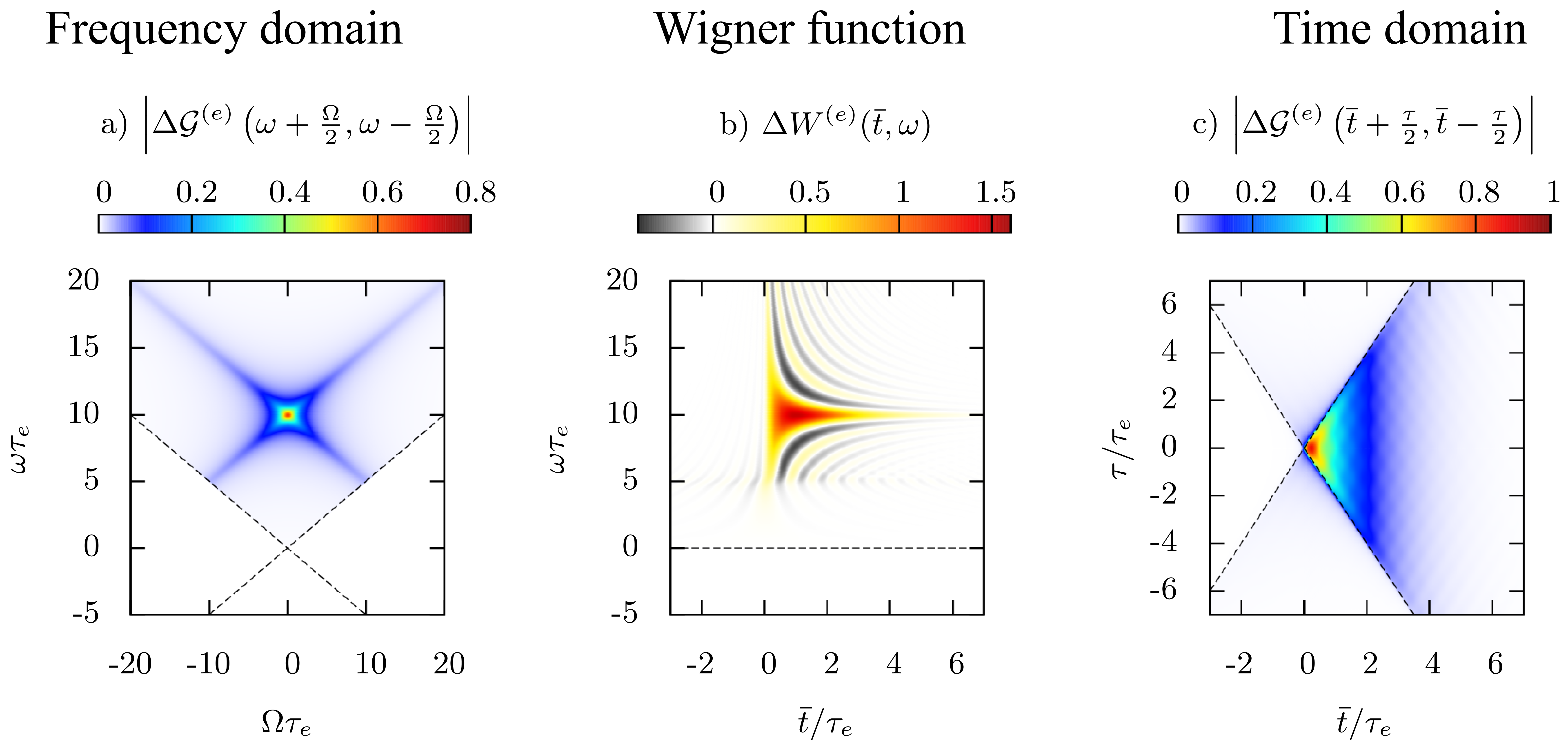
In general: $\mathcal{G}^{(e)}(t,t') = \mathcal{G}^{(e)}_{F}(t,t') + \Delta\mathcal{G}^{(e)}(t,t')$

Fermi sea contribution — Excess single electron coherence

Single electron coherence

Three different representations

$$\varphi_e(\omega) = \frac{\mathcal{N}_e \Theta(\omega)}{\omega - \omega_\mathrm{e} - \mathrm{i}/2\tau_\mathrm{e}}$$



D. Ferraro *et al*, Phys. Rev. B **88**, 205303 (2013)

The electronic Wigner function

Definition :

$$W^{(e)}_{\rho,x}(t,\omega) = v_F \int_{\mathbb{R}} \mathrm{e}^{\mathrm{i}\omega\tau} \mathcal{G}^{(e)}_{\rho,x}\left(t+\frac{\tau}{2}\middle|t-\frac{\tau}{2}\right)\,\mathrm{d}\tau$$

Marginals :

$$\langle i(x,t)\rangle_\rho = -e\int_{\mathbb{R}} \Delta W^{(e)}_{\rho,x}(t,\omega)\,\frac{\mathrm{d}\omega}{2\pi}$$

$$f_e(\omega|\rho,x) = \overline{W^{(e)}_{\rho,x}(t,\omega)}^{\,t}$$

Sinusoidal current

Single electron coherence: $\mathcal{G}^{(e)}(t,t') = \mathcal{G}^{(e)}_{\mu,T_{\text{el}}}(t,t')\,\exp\left(\frac{ie}{\hbar}\int_t^{t'} V(\tau)\,d\tau\right)$

AC current: sinusoidal $V(t) = V\cos(2\pi f t)$

Parameters

$k_B T_e/hf$ # of thermal photons at *hf*

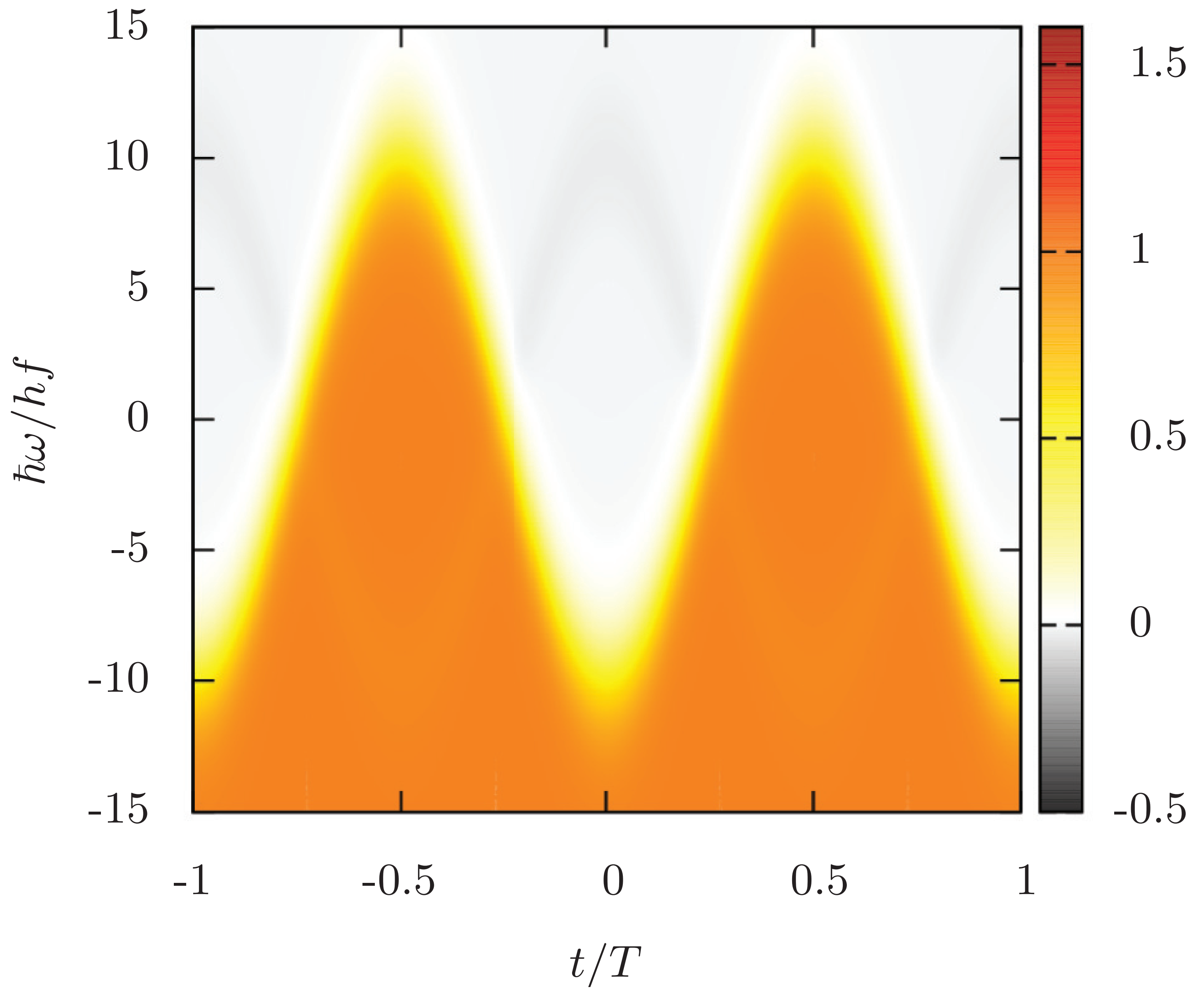
eV/hf # of photons at *hf* or emitted charge per period (in units of *e*)

$$W^{(e)}(\omega,t) = \sum_{n=-\infty}^{+\infty} \frac{J_n(\frac{2eV}{hf}\cos(2\pi f t))}{e^{\beta_{\text{el}}(\hbar\omega + nhf)}+1}$$

Sinusoidal current

Large amplitude and high temperature

$$eV \gg hf \qquad k_B T_{\text{el}} \gg hf$$

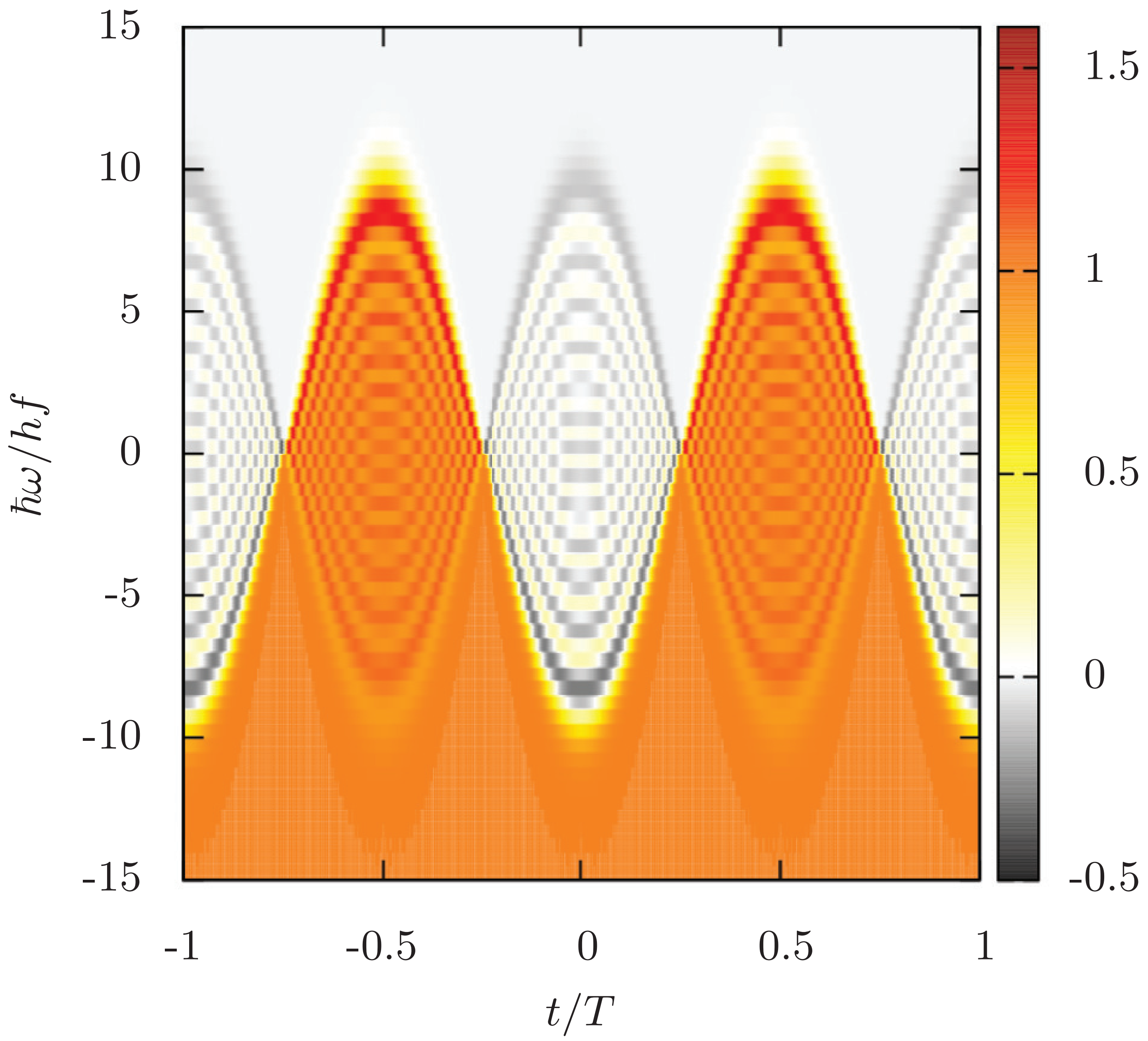


Agitated Fermi sea: $W^{(e)}(t,\omega) = f_{\mu(t),T_{\text{el}}}(\omega)$

$$\mu(t) = -eV_d(t)$$

Large amplitude and zero temperature

$$eV \gg hf \qquad T_{\text{el}} = 0\ \text{K}$$

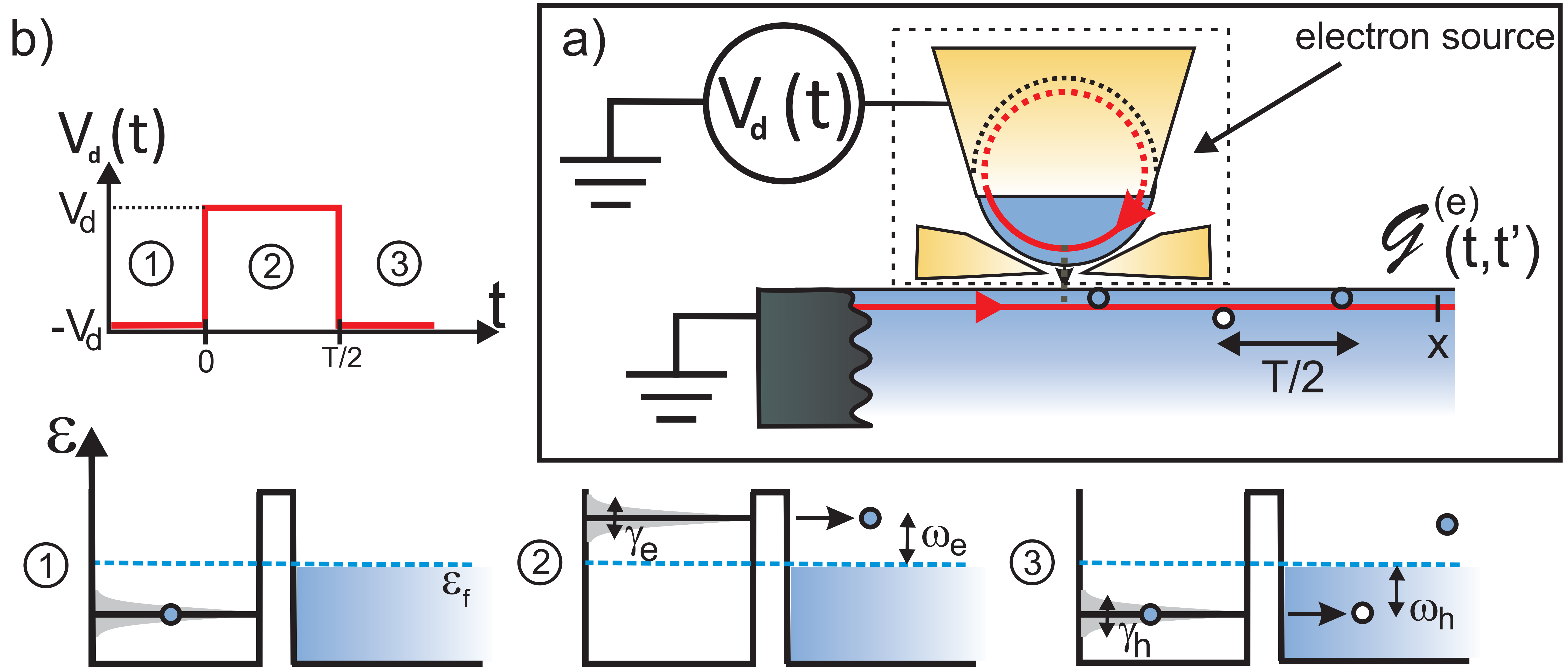


Quantum ripples:

$$\int_{t-\tau/2}^{t+\tau/2} V(\tau')\,d\tau' = \tau V(t) + \frac{V''(t)}{24}\tau^3 + \dots$$

Quantum RC circuit

J. Gabelli *et al*,
Science **313**, 499 (2006)



G. Fève *et al*,
Science **316**, 1169 (2007)

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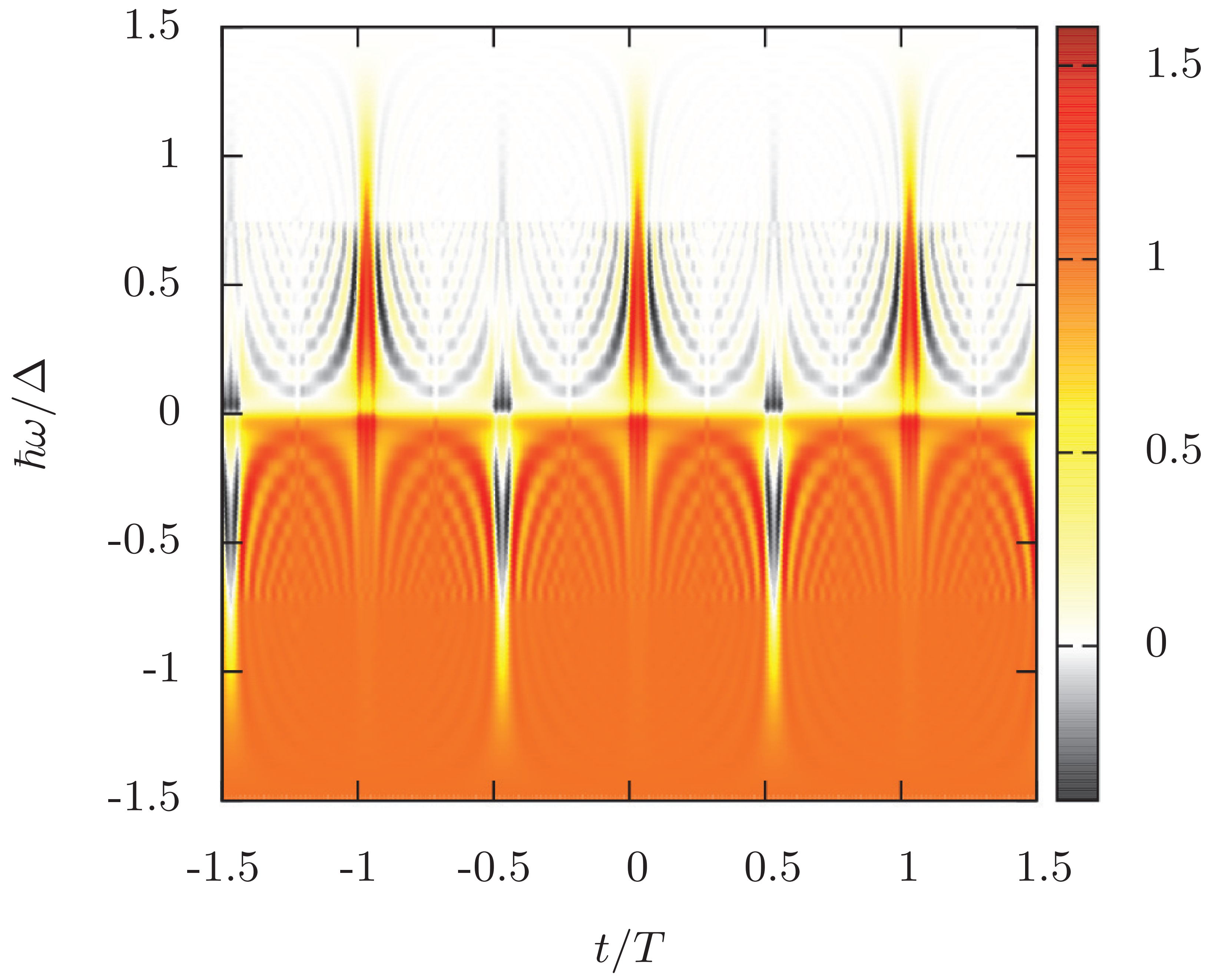
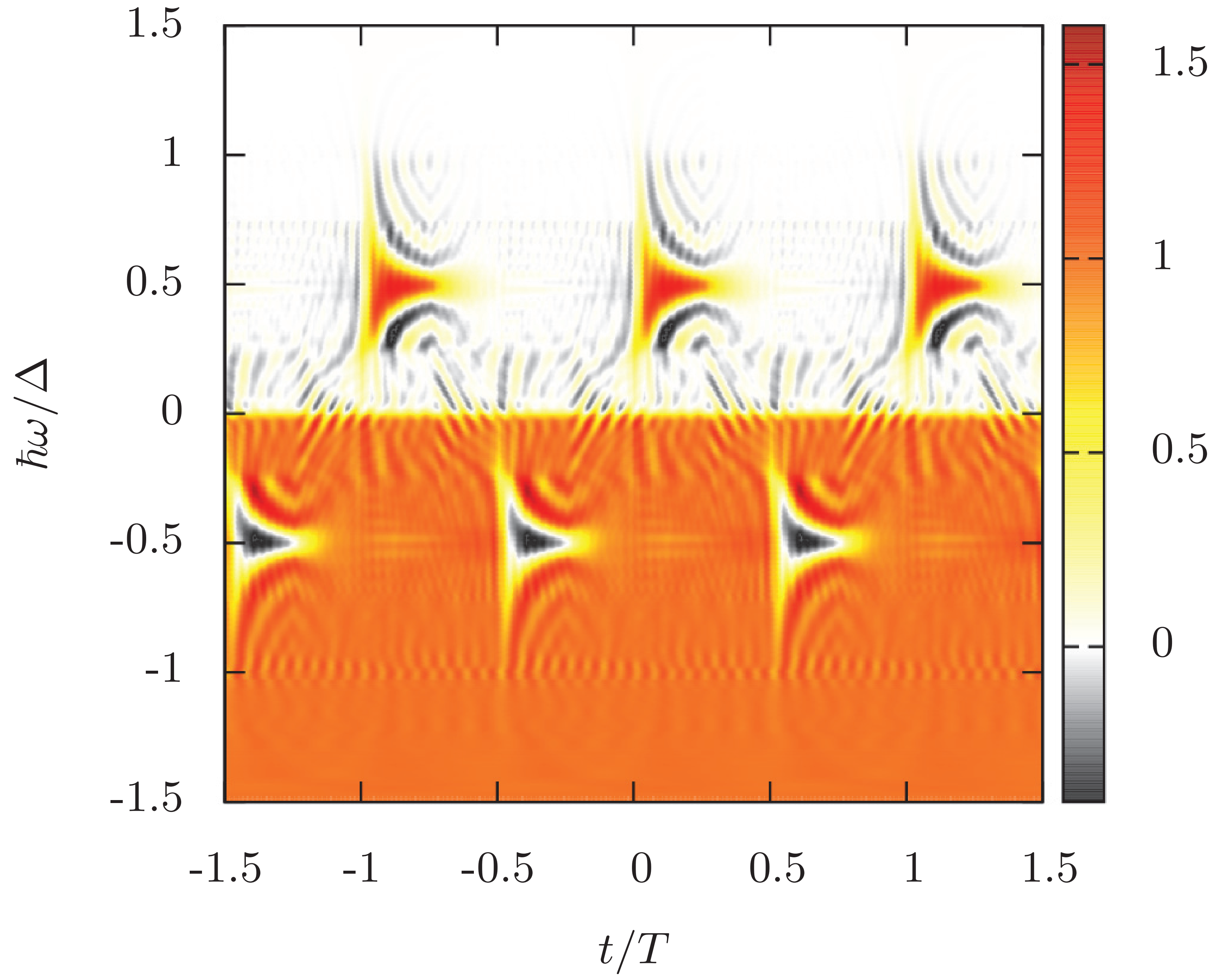
Independent particle computation (Floquet scattering theory)

$D = 1$ $\qquad$ $D \sim 0.4$





Energy resolved e and h excitations

Mach Zehnder interferometry

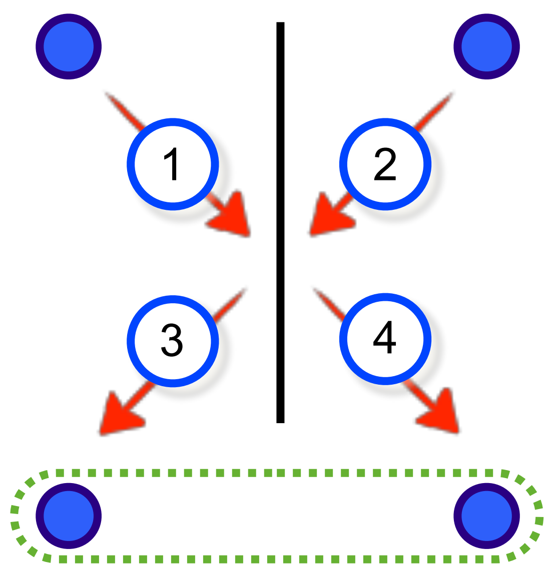
$$\Delta\mathcal{G}^{(e)} \longrightarrow \boxed{\text{MZI}} \longrightarrow \langle i_{\text{dc}}^{(\text{out})}\rangle$$

linear filtering

But strong decoherence issues :

S. Tewari *et al*, Phys. Rev. B **93**, 035420 (2016)

HOM interferometry



$$\Delta\mathcal{G}_1^{(e)},\ \Delta\mathcal{G}_2^{(e)} \longrightarrow \boxed{\text{HOM}} \longrightarrow S_i^{(\text{out})}(\omega=0)$$

overlap

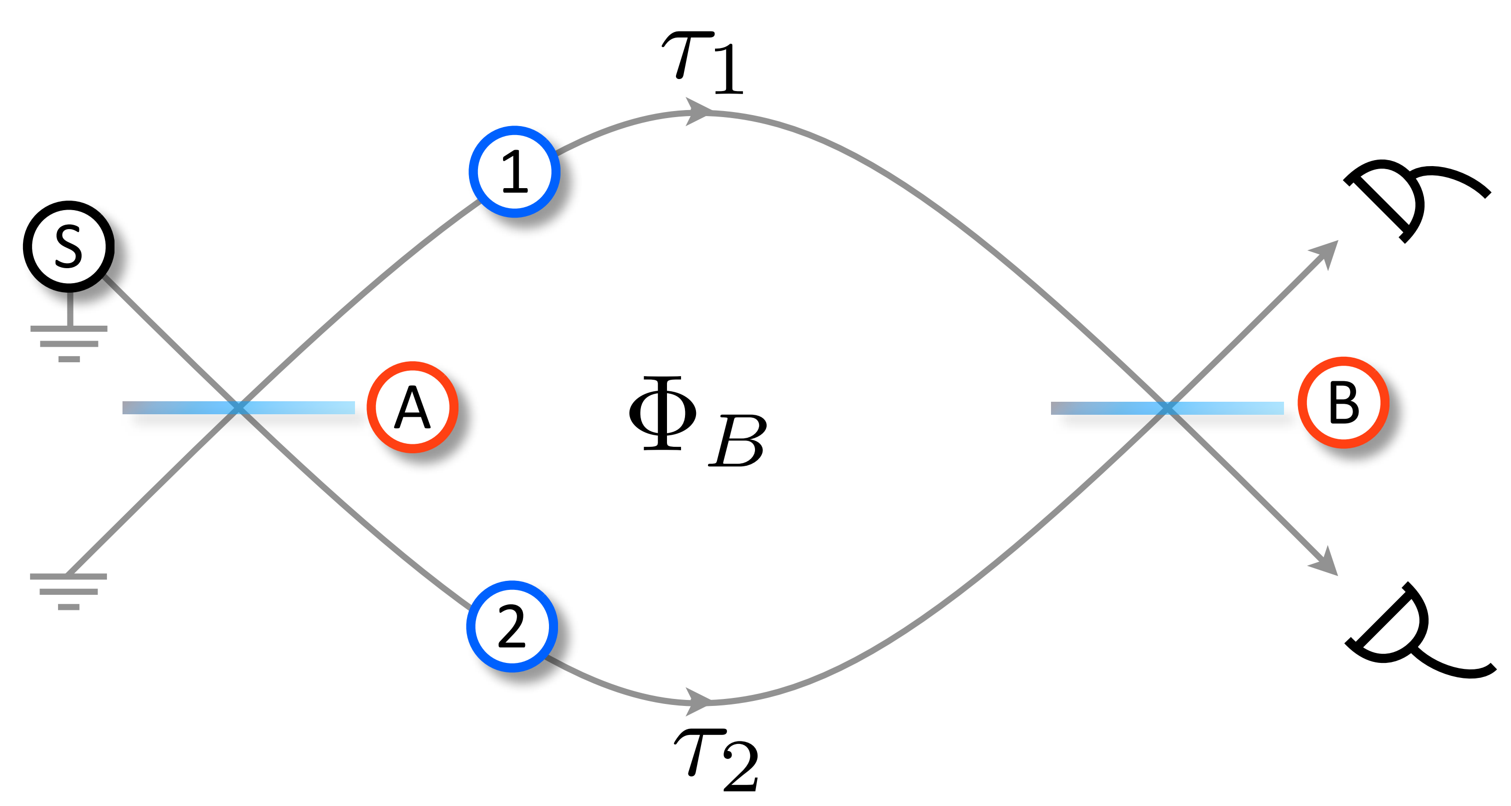
E. Bocquillon *et al*, Science **339**, 1054 (2013)

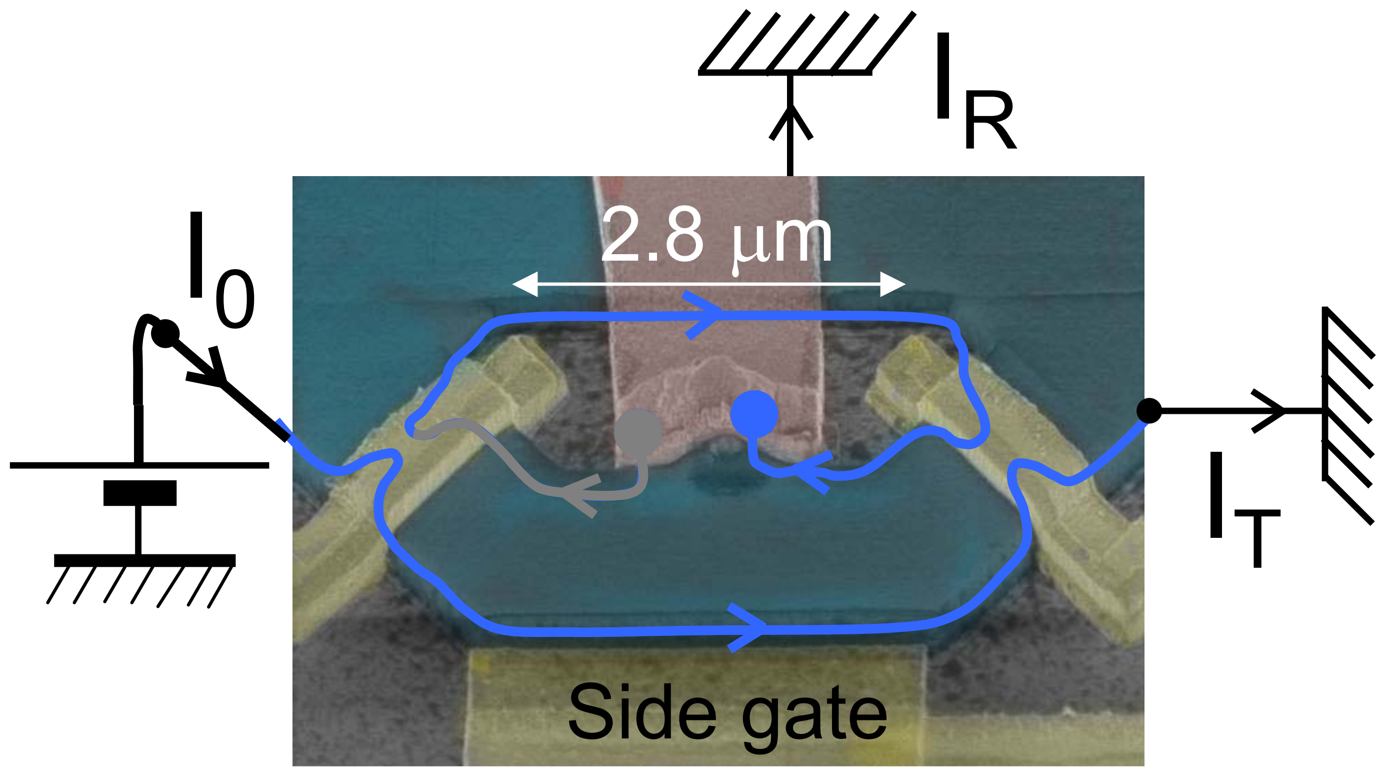
Quantum tomography: C. Grenier *et al*, New Journal of Physics **13**, 093007 (2011)

T. Jullien *et al,* Nature **514**, 603-607 (2014)

Decoherence studies: A. Marguerite *et al,* Phys. Rev. B **94**, 115311 (2016)

Mach Zehnder interferometry





Courtesy P. Roche

$$\Delta W_{1,\mathrm{out}}(t,\omega) = \mathcal{M}_{1,1}\Delta W_S^{(e)}(t-\tau_1,\omega) + \mathcal{M}_{2,2}\Delta W_S^{(e)}(t-\tau_2,\omega)$$

Classical contributions

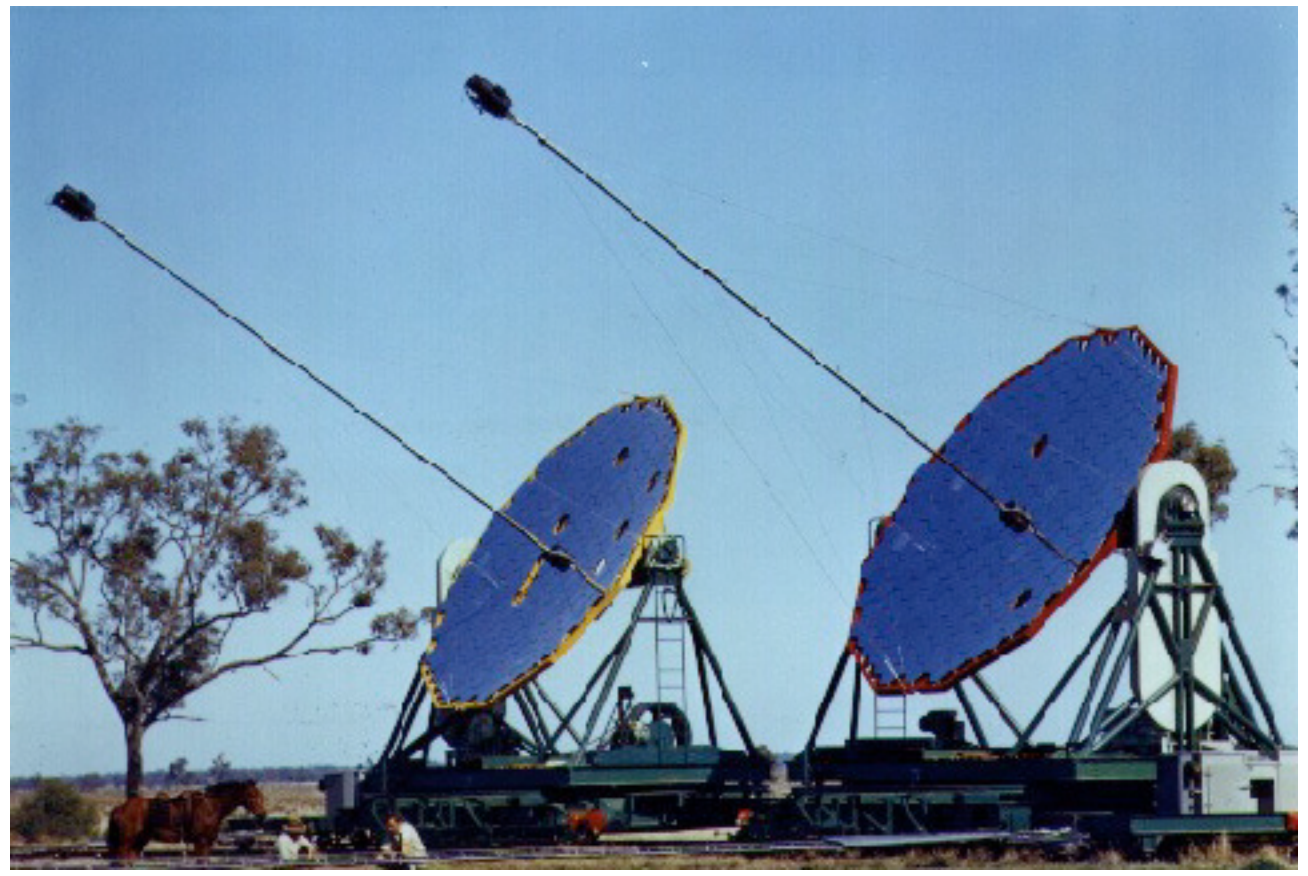
$$+2|\mathcal{M}_{1,2}|\,\cos(\omega(\tau_1-\tau_2)+\phi)\,\Delta W_S^{(e)}\left(t-\frac{\tau_1+\tau_2}{2},\omega\right)$$

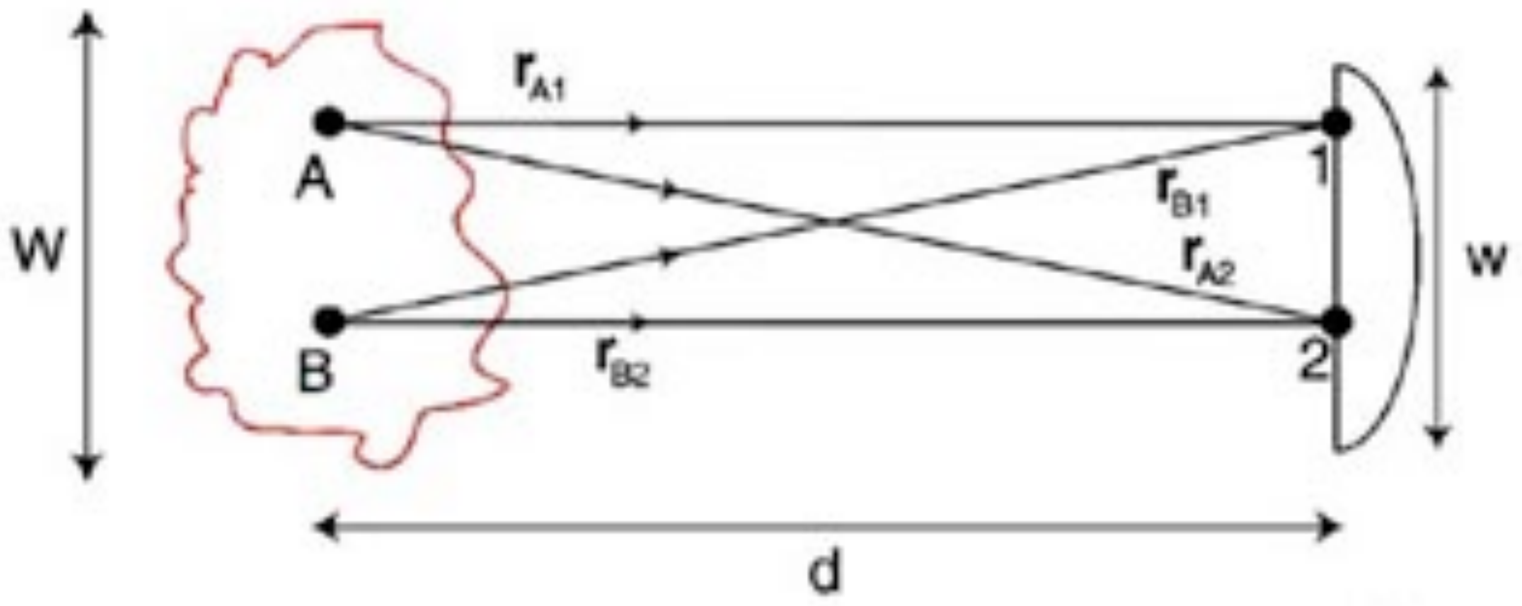
Quantum contributions

D. Ferraro *et al*, Phys. Rev. B **88**, 205303 (2013)

Time domain: G. Haack, M. Moskalets et M. Büttiker, Phys. Rev. B **84**, 081303 (2011)

Single electron tomography





$$P = \left| \psi_A e^{i\phi_{A1}} \psi_B e^{i\phi_{B2}} \pm \psi_A e^{i\phi_{A2}} \psi_B e^{i\phi_{B1}} \right|^2$$

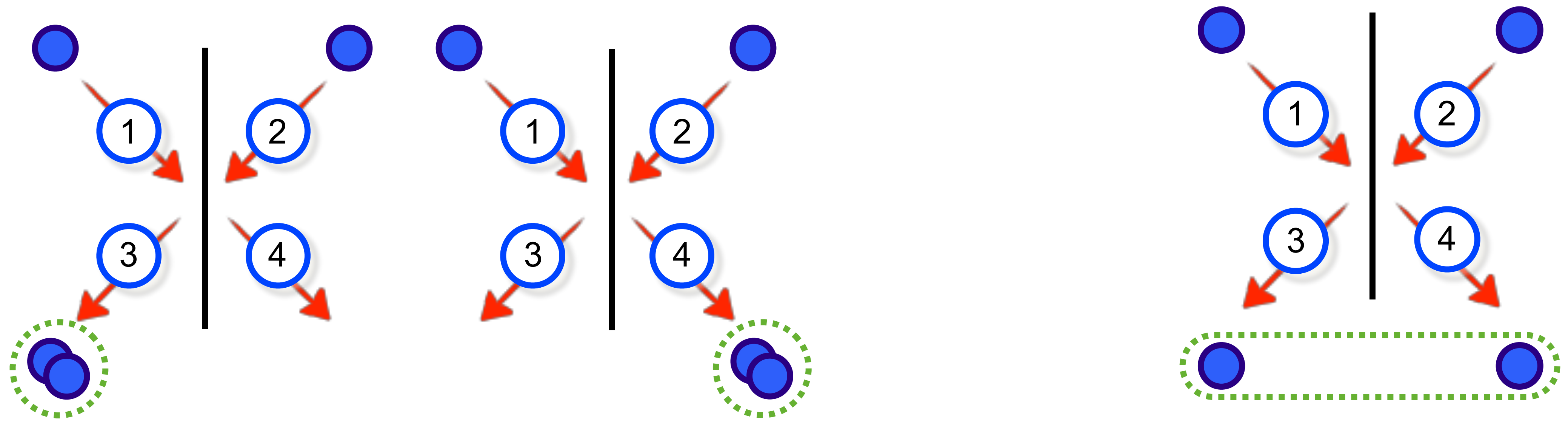
Nature **178**, 1046 (1956)

Two particle interference interpretation

U. Fano, Am. J. Phys. **29**, 539 (1961)

Undistinguishable **bosons**

Fermions



Bunching

Anti-bunching

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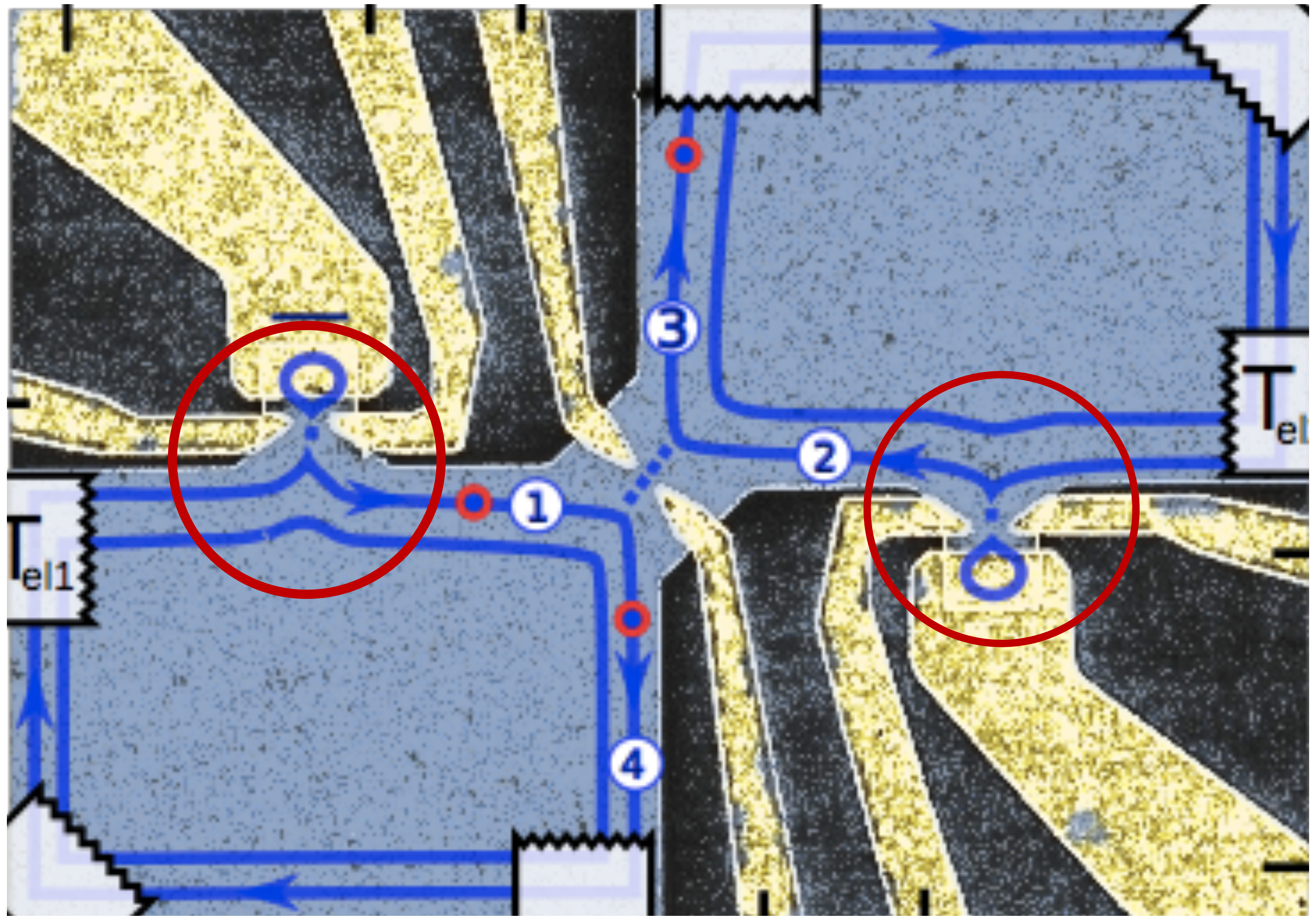
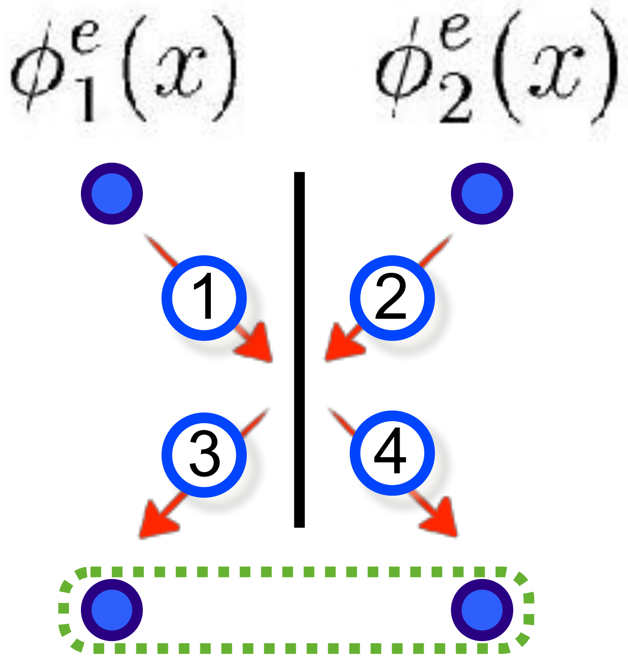
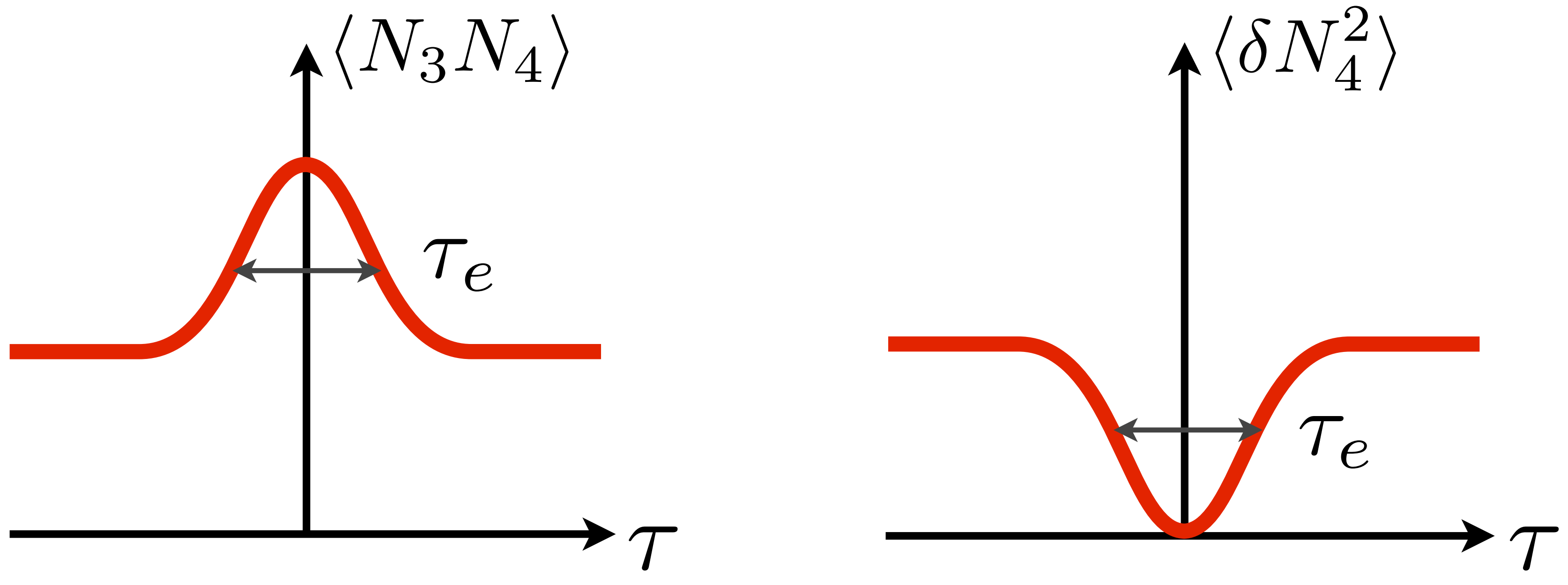
Single electron emitter #1



Single electron emitter #2



$$P(\text{antibunching}) = \frac{1}{2}\left[1 + |\langle\phi_1^e|\phi_2^e\rangle|^2\right]$$



E. Bocquillon et al, Science **339**, 1054 (2013)

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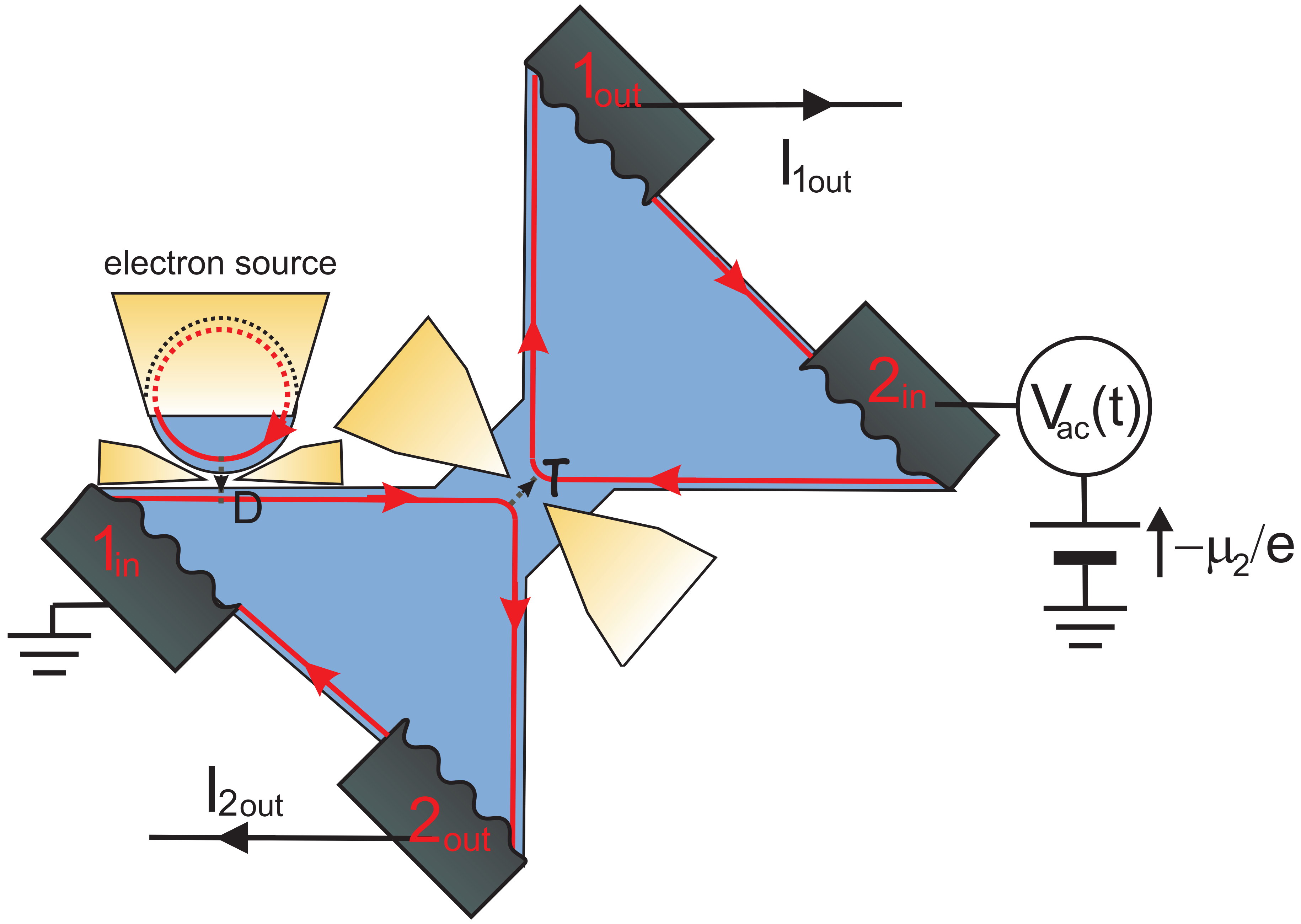


Current noise measurements

$$S_{11}^{(S_1,S_2)} = S_{11}^{(S_1)} + S_{11}^{(S_2)} + \Delta S_{11}^{(\mathrm{HOM})}$$

$$\Delta S_{11}^{(\mathrm{HOM})} = -e^2 \int \overline{(\Delta W_{S_1}^{(e)}\, \Delta W_{S_2}^{(e)})(t,\omega)}^{t} \,\frac{d\omega}{2\pi}$$

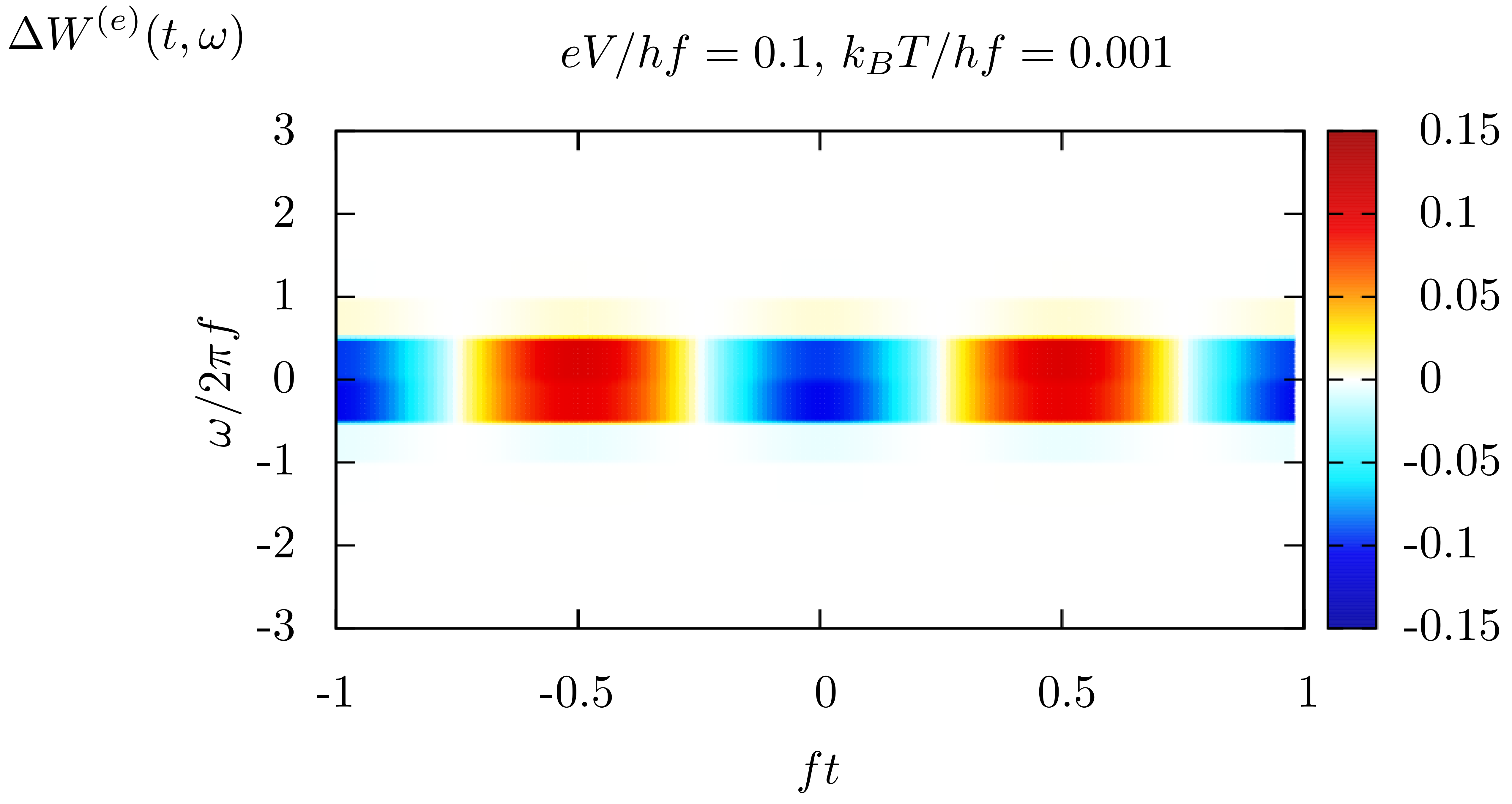
«*The noise is the signal* » (R. Landauer 1998)

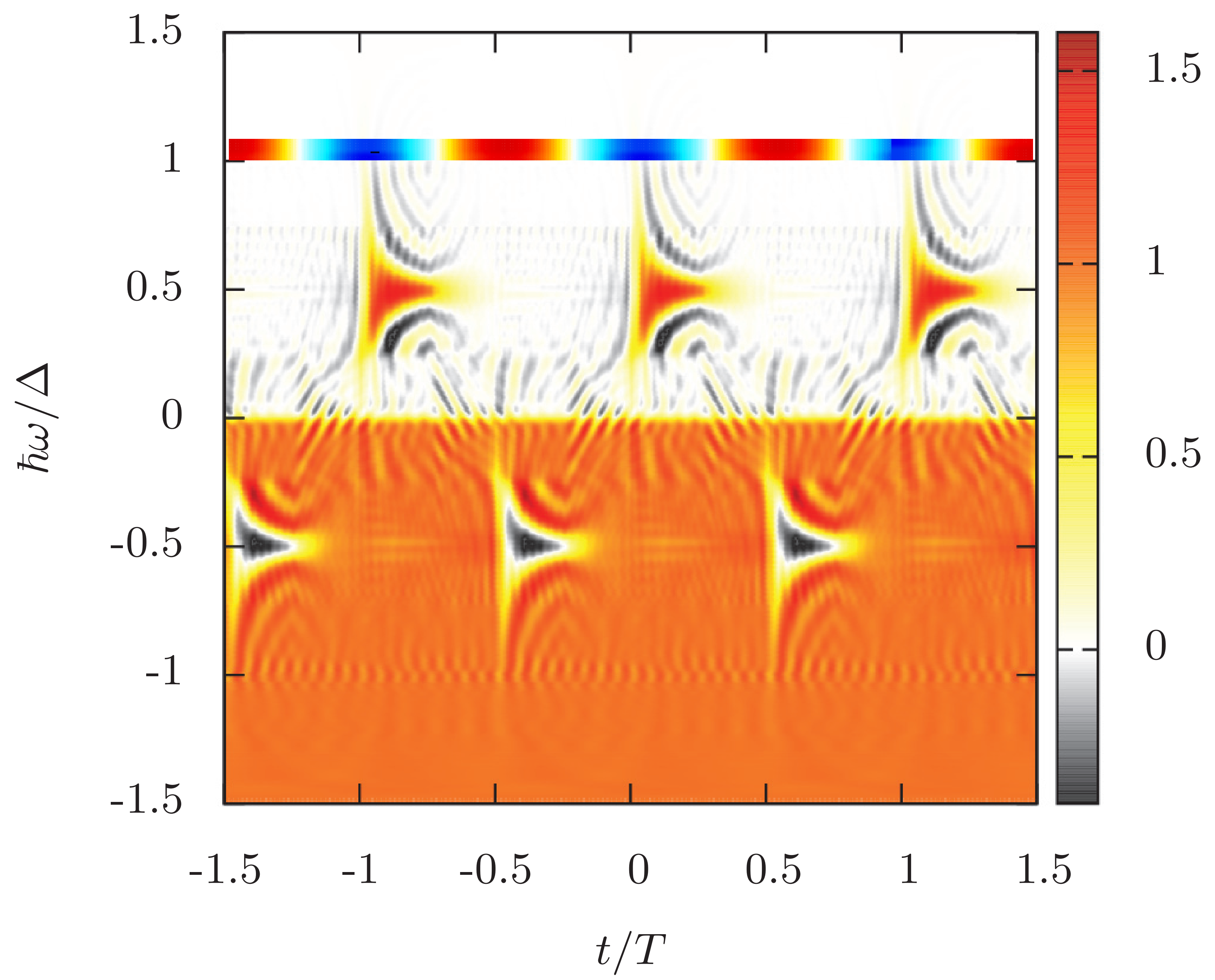
C. Grenier *et al*, New Journal of Physics **13**, 093007 (2011)

D. Ferraro *et al*, Phys. Rev. B **88**, 205303 (2013)

Single electron tomography in a nutshell

Excess Wigner function of a small ac drive $V_{\mathrm{ac}}(t) = V\cos(2\pi f t)$





Variant implémentation: T. Jullien *et al,* Nature **514**, 603-607 (2014)

Two-electron coherence

ENS Lyon

2e-coherence: $\mathcal{G}_\rho^{(2e)}(1,2|1',2') = \mathrm{Tr}\left(\psi(2)\psi(1)\rho\,\psi^\dagger(1')\psi^\dagger(2')\right)$

M. Moskalets, Phys. Rev. B. **89**, 045402 (2012)

- Encodes two-electron wave-functions
- Symmetries in 4D space: quantum statistics

$$\prod_{k=1}^{N} \psi^\dagger[\varphi_k]\,|\emptyset\rangle \ \text{ with } \ \langle\varphi_k|\varphi_l\rangle = \delta_{k,l}$$

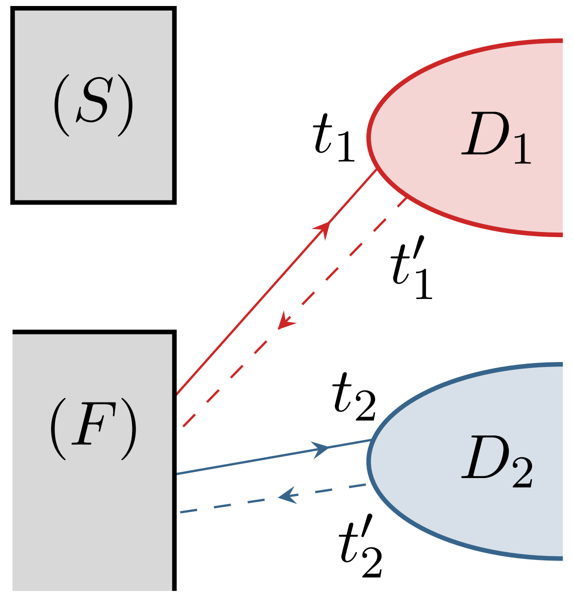
$$\mathcal{G}^{(2e)}(1,2|1',2') = \sum_{\{k,l\}} \varphi_{k,l}(1,2)\,\varphi^*_{k,l}(1',2')$$

where $\varphi_{k,l}(x,y) = \varphi_k(x)\varphi_l(y) - \varphi_k(y)\varphi_l(x)$

Question:

- Intrinsic contribution of the source to two-electron coherence?

Intrinsic two electron coherence



$$\mathcal{G}_{\rho}^{(2e)}(1,2|1',2') = \mathcal{G}_{F}^{(2e)}(1,2|1'2')$$

E. Thibierge *et al*, Phys. Rev. B. **93**, 081302(R) (2016)

Current noise measurement

A. Mahé *et al*, Phys. Rev. B **82**, 201309 (2010)
F. Parmentier *et al*, Phys. Rev. B **85**, 165438 (2012)

Direct noise measurement:

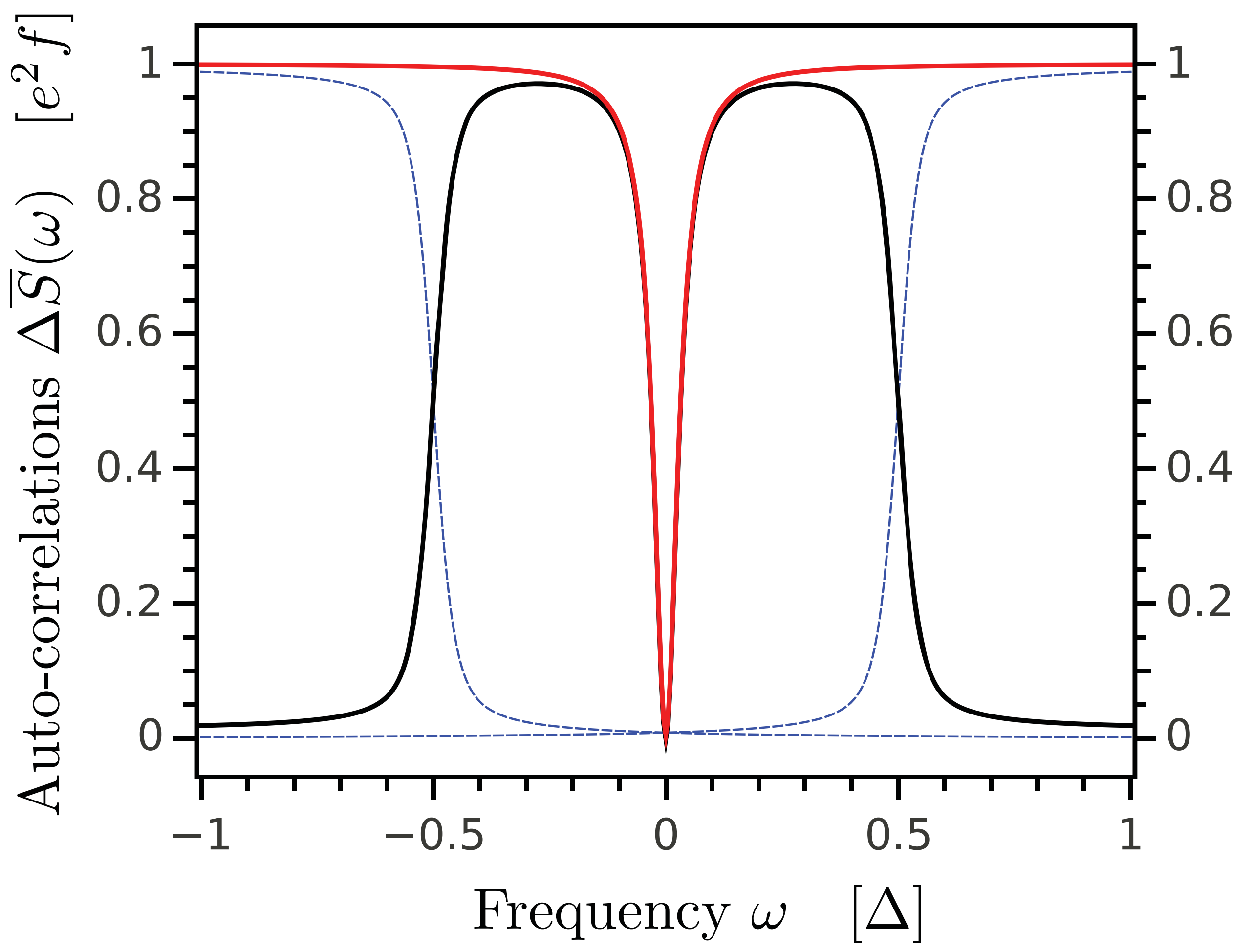
S → $i(t)$

$$S_i(t,t') = \langle i(t)\, i(t')\rangle - \langle i(t)\rangle\langle i(t')\rangle$$

$$\Delta S_i(t,t') = S_i(t,t')_{\mathrm{on}} - S_i(t,t')_{\mathrm{off}}$$

Noise spectrum: $$\Delta\overline{S}(\omega) = \int \overline{\Delta S_i(t+\tau/2, t-\tau/2)}^{\,t}\, e^{i\omega\tau}\,\mathrm{d}\tau$$

Noise spectrum of the mesoscopic capacitor

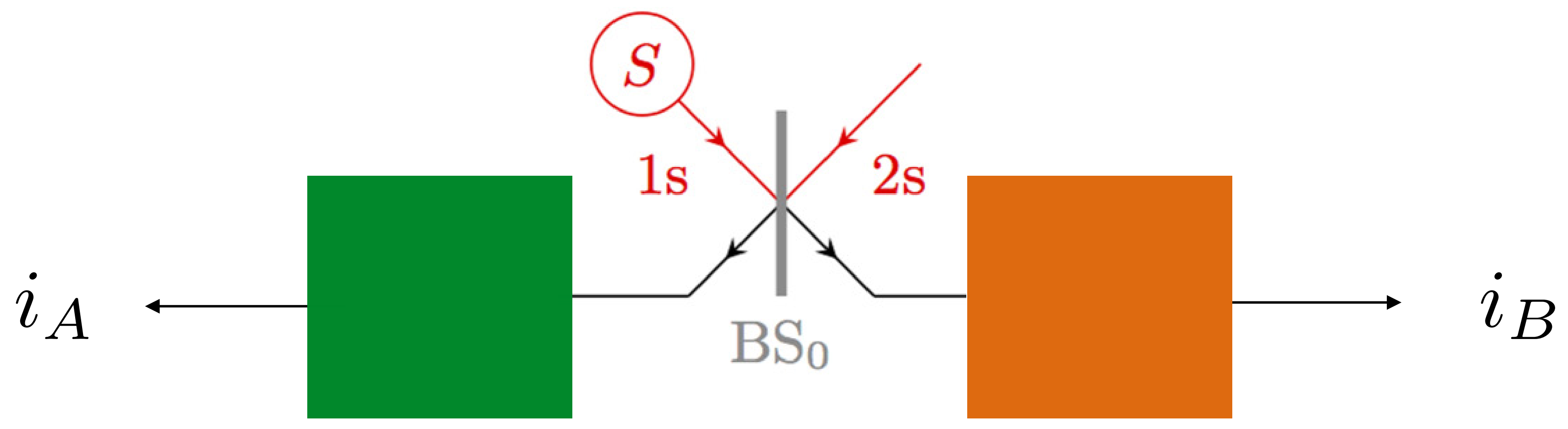


Current noise from electronic coherences

$$\Delta S_i(t,t') = -e\langle i(t)\rangle_S\,\delta(t-t') + (ev_F)^2\left(\Delta\mathcal{G}_S^{(2e)}(t,t'|t,t') - \Delta\mathcal{G}_S^{(e)}(t|t)\Delta\mathcal{G}_S^{(e)}(t'|t')\right)$$
$$- (ev_F)^2\left(\mathcal{G}_F^{(e)}(t|t')\Delta\mathcal{G}_S^{(e)}(t|t') + \mathcal{G}_F^{(e)}(t'|t)\Delta\mathcal{G}_S^{(e)}(t'|t)\right)$$

B. Roussel *et al*, Physica Status Solidi B **254**, 1600621 (2017)

Accessing two electron coherence



A-detector
Signal: outgoing currents

B-detector
Signal: outgoing currents

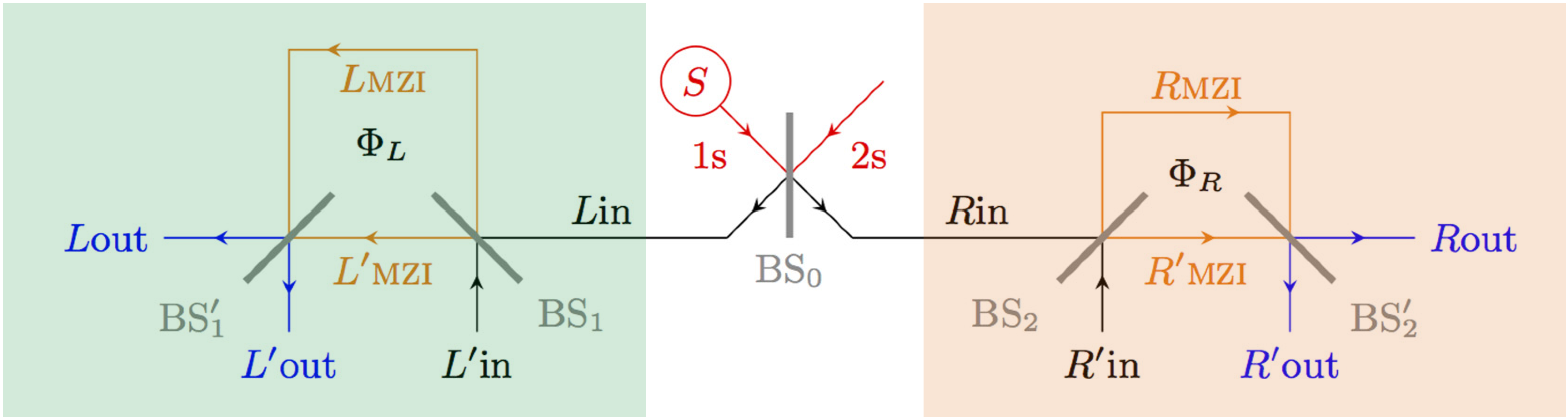
Two particle interferences at the beam splitter: $\Delta\mathcal{G}^{(2e)}_{\mathrm{out}_{\mathrm{BS}_0}}(1\,t_1;2\,t_2|1\,t_1';2\,t_2') = RT\,\Delta\mathcal{G}^{(2e)}_{S}(t_1,t_2|t_1',t_2')$

Current correlations after the detectors: $\langle i_A\, i_B\rangle = \left(\mathcal{L}^{(1)}_A \otimes \mathcal{L}^{(2)}_B\right)\left[\Delta\mathcal{G}^{(2e)}_{\mathrm{out}_{\mathrm{BS}_0}}(1\,t_1;2\,t_2|1\,t_1';2\,t_2')\right]$

It combines:

- HBT interferometry: partitioning of two-electron coherence at a beam splitter
- Single particle interferometry: converting off-diagonal single-electron coherence into measurable signal

Example: Franson interferometry



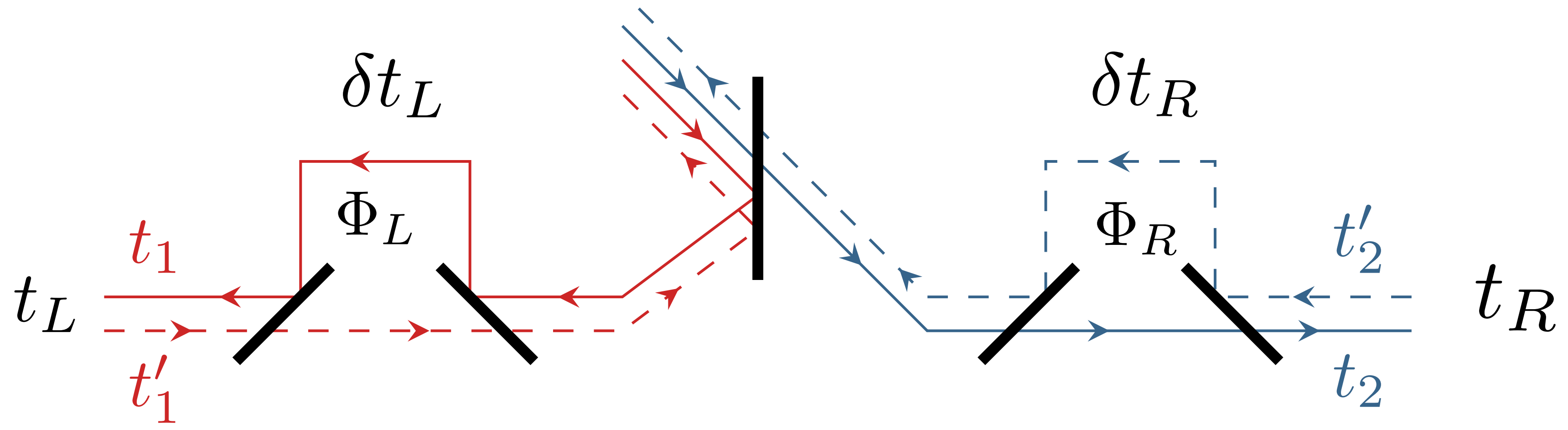
A-detector

Signal: outgoing currents
Parameters: time of flights, **AB flux**

B-detector

Signal: outgoing currents
Parameters: time of flights, **AB flux**

The Franson signal: current correlations between left/right detectors with both flux sensitivities

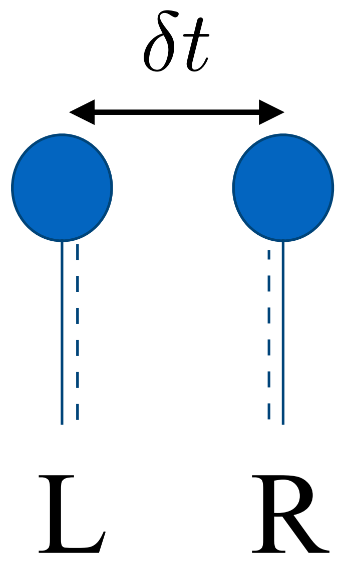


$$\sim (ev_F)^2 e^{-i(\Phi_L+\Phi_R)} \Delta\mathcal{G}_S^{(2e)}(t_L, t_R | t_L - \delta t_L, t_R - \delta t_R)$$

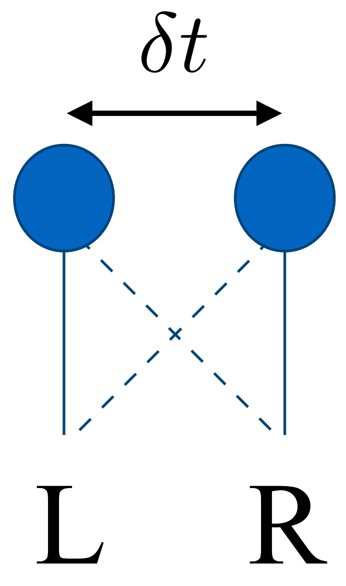
Franson signals

An electron pair

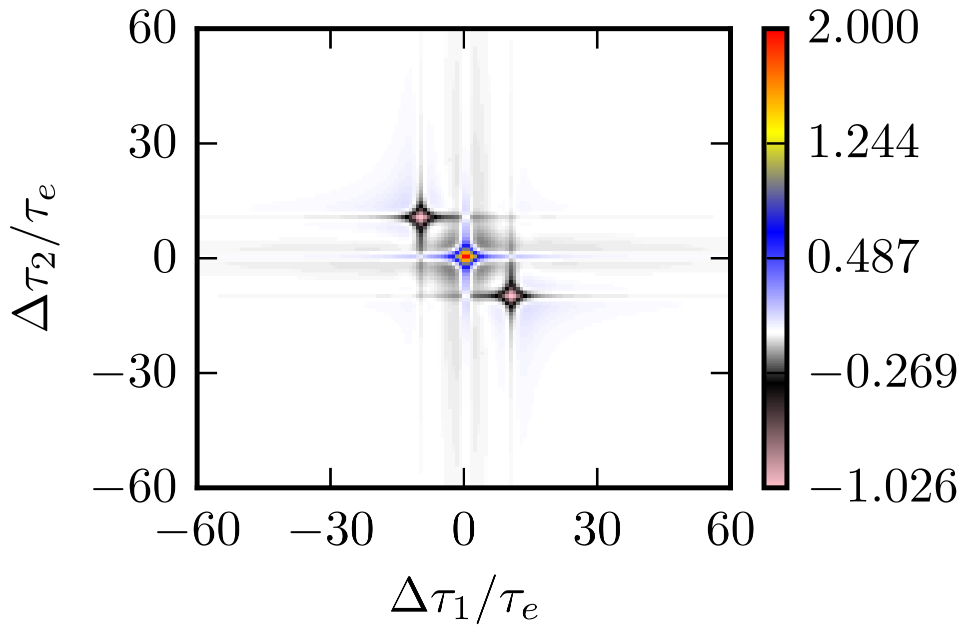
Two Levitons separated by 10x their width τ_0



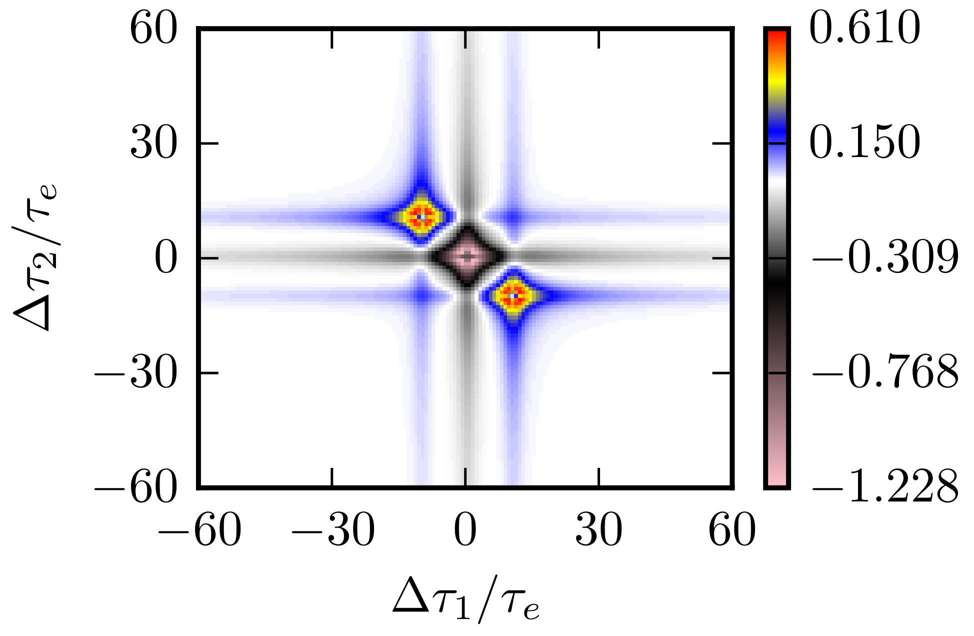
$\Delta\tau_1 \simeq -\Delta\tau_2 \simeq \tau_0$



$\Delta\tau_1 \simeq -\Delta\tau_2 \simeq \delta t$



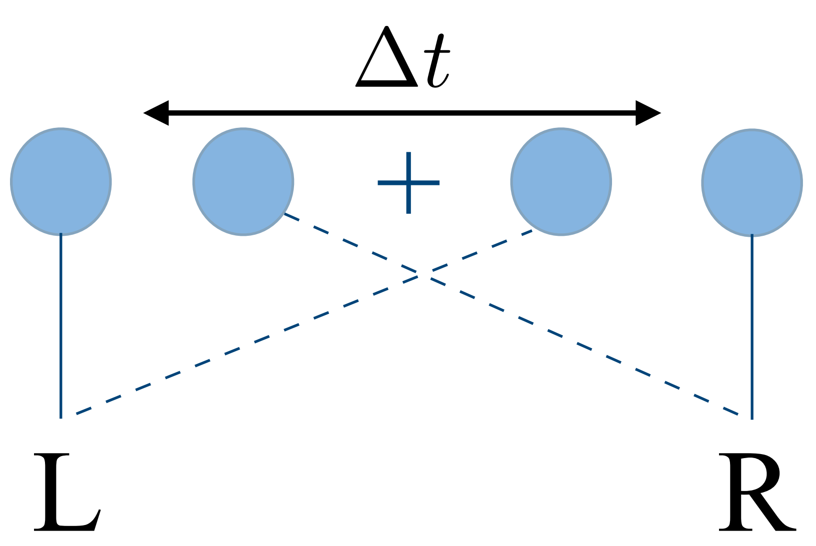
Real part



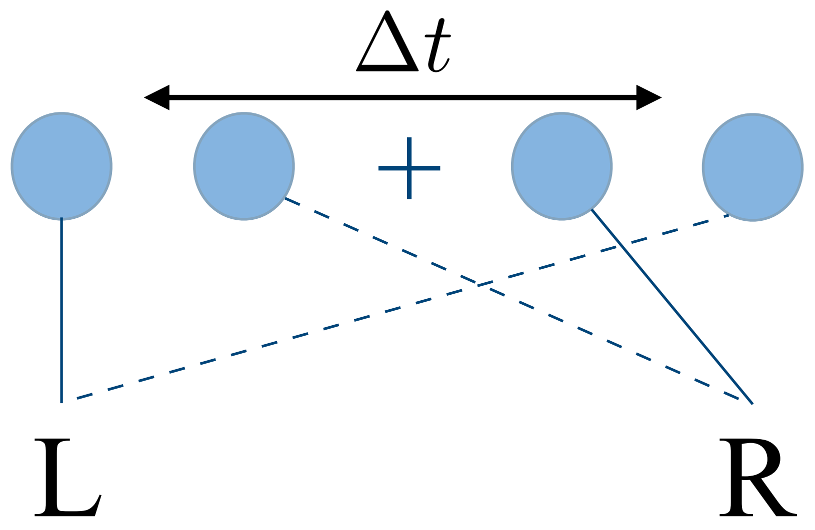
Imaginary part

A time-bin entangled electron pair

Quantum superposition of two pairs separated by 30x their width

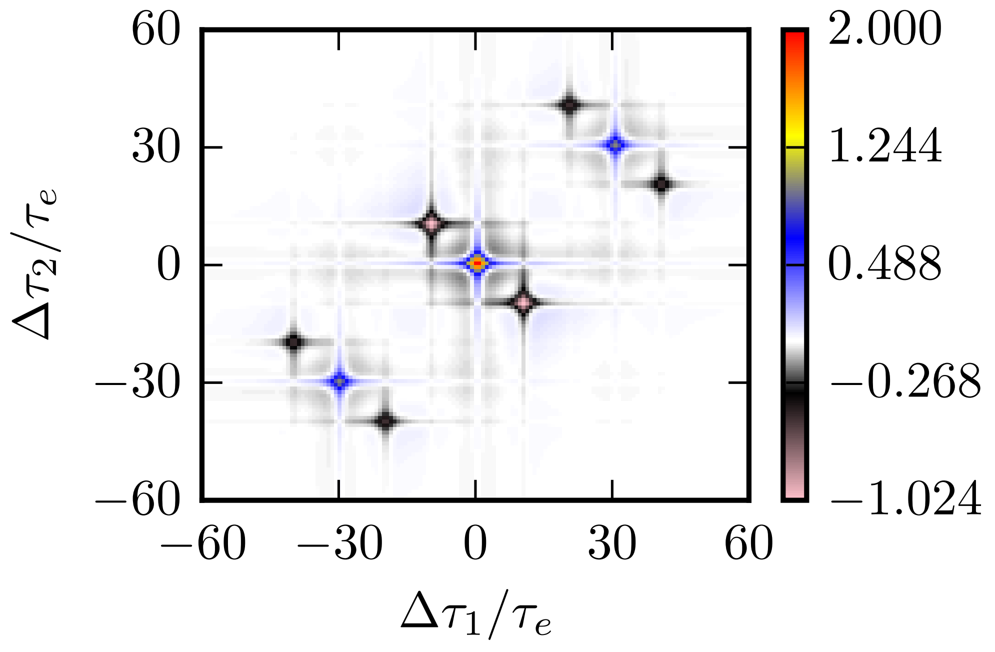


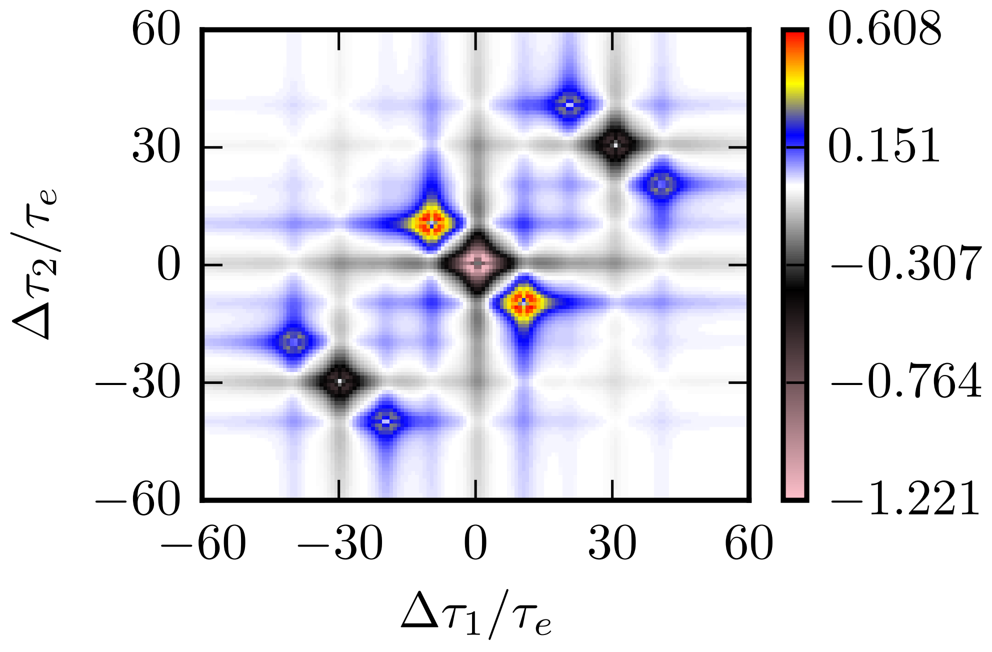
$\Delta\tau_1 \simeq \Delta\tau_2 \simeq \Delta t$



$\Delta\tau_1 \simeq \Delta t + \delta t$

$\Delta\tau_2 \simeq \Delta t - \delta t$





E. Thibierge *et al*, Phys. Rev. B. **93**, 081302(R) (2016)

Take home message #2

What are the "(quantum) signals" carried by electrical currents (*in a metal*) ?

Classical signal NONE !

Quantum signals

$$\mathcal{G}_\rho^{(e)}(x,t|x',t') = \mathrm{Tr}(\psi(x,t)\,\rho\,\psi^\dagger(x',t'))$$

$$\mathcal{G}_\rho^{(2e)}(1,2|1',2') = \mathrm{Tr}\left(\psi(2)\psi(1)\rho\,\psi^\dagger(1')\psi^\dagger(2')\right)$$

Higher order coherence: information on the full charge statistics…

Problem: really hard to access experimentally…

Take home message #3

Electron quantum optics experiments realize on-chip « quantum signal processing »

Mach-Zehnder interferometry	$\Delta\mathcal{G}^{(e)}$	MZI	$\langle i_{dc}^{(out)}\rangle$	*linear filtering*
Hong Ou Mandel interferometry	$\Delta\mathcal{G}_1^{(e)}$, $\Delta\mathcal{G}_2^{(e)}$	HOM	$S_i^{(out)}(\omega = 0)$	*overlap*
Franson interferometry	$\Delta\mathcal{G}^{(2e)}$	Fr.I	$\Delta S_{12}^{(out)}$	*linear filtering*

E. Thibierge *et al*, Phys. Rev. B **93**, 081302 (2016)
B. Roussel *et al*, Physica Status Solidi B **254**, 1600621 (2017)

Perspectives

From electronic coherences to quantum information quantities: quantitative criteria for *2e* entanglement ?

Plan

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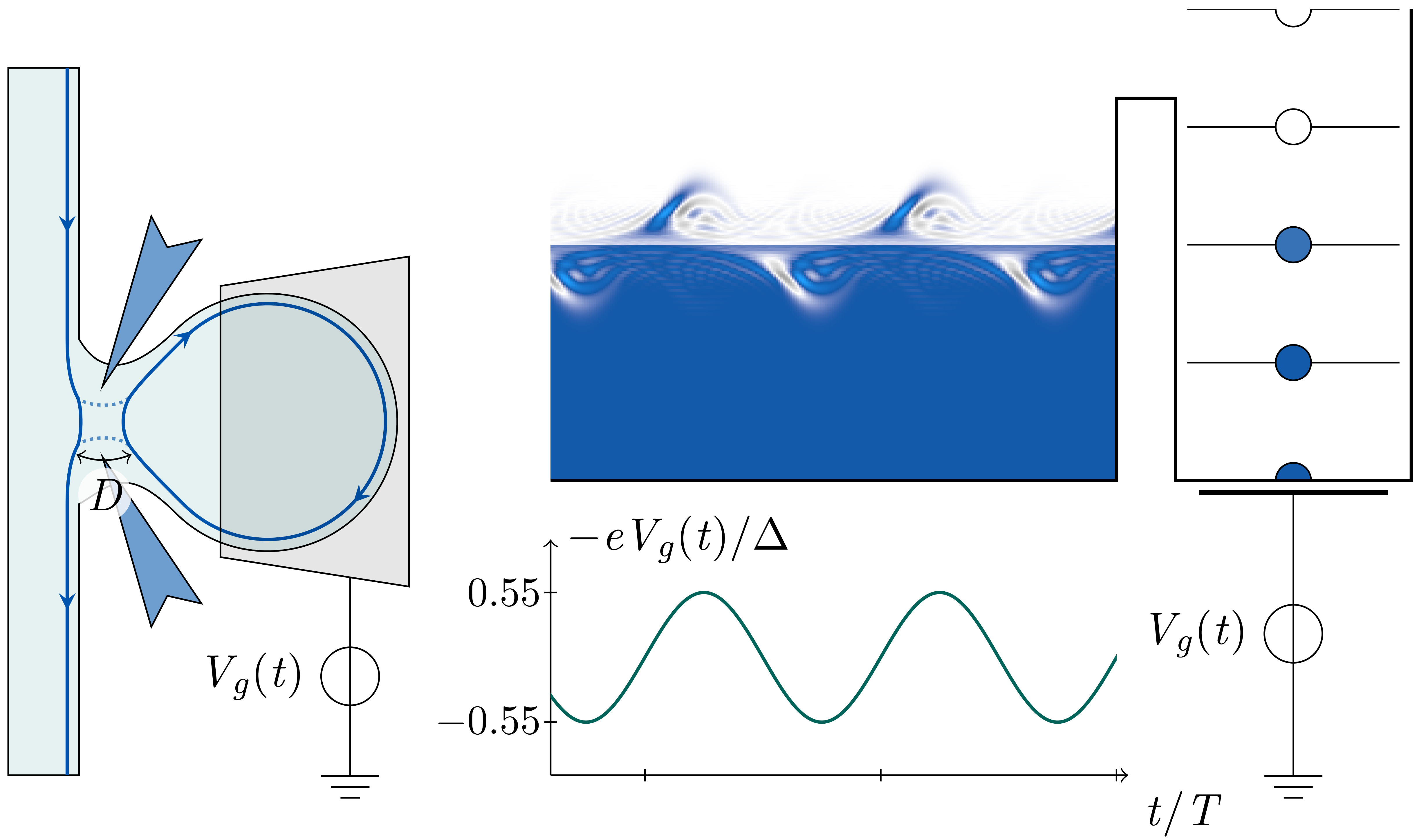
- Introduction
- Lessons from quantum optics
- Electron quantum optics
- Signal processing for quantum electrical currents
- Conclusion & perspectives

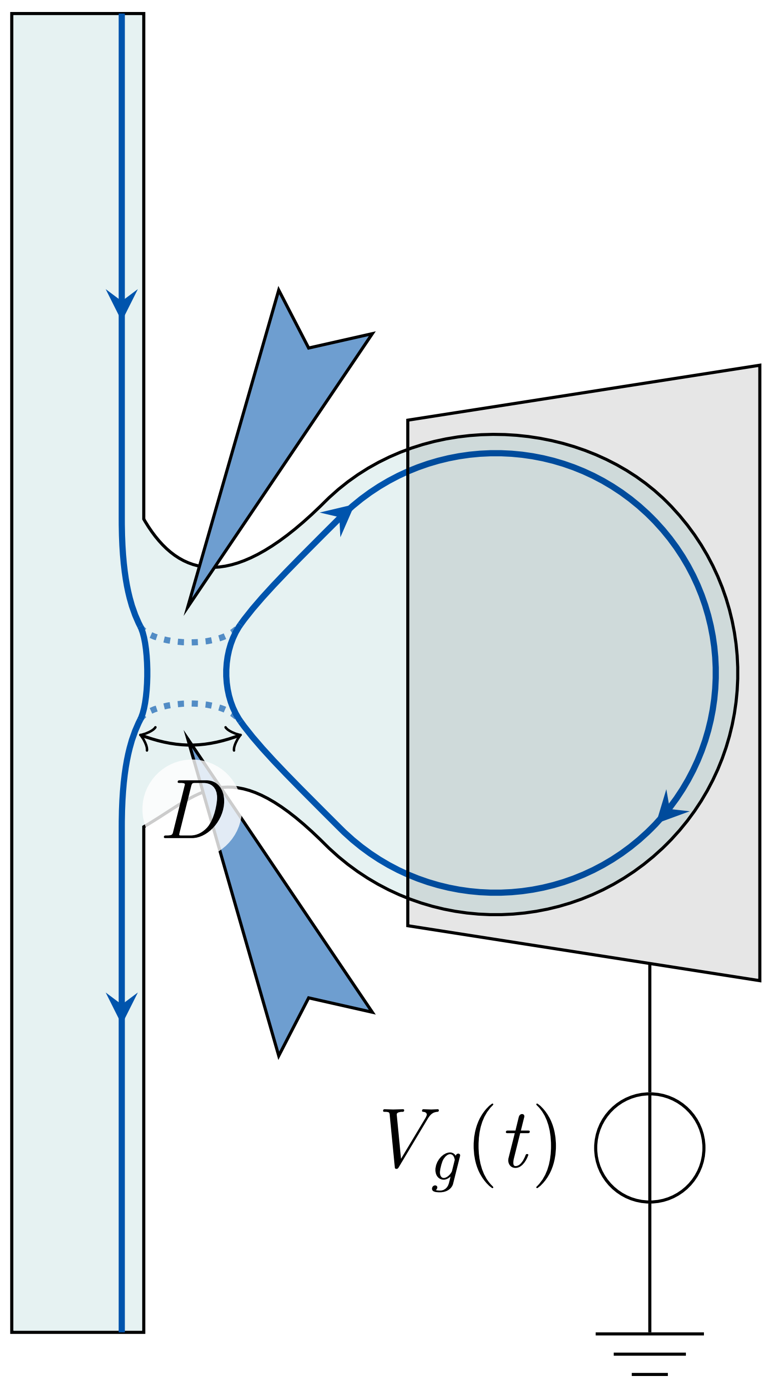
Autopsy of a quantum electrical current ?

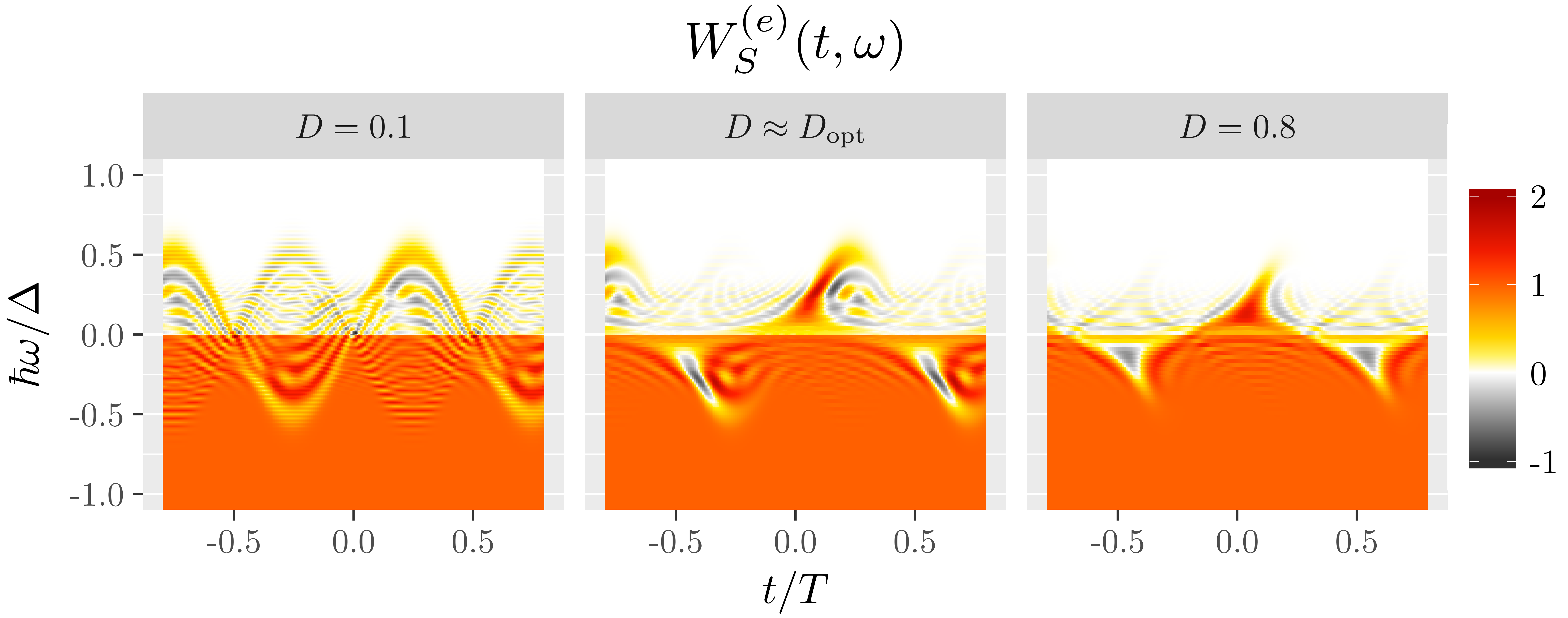


What are the single electron wave functions contained in this electrical current ?

B. Roussel, PhD thesis (tel-01730943)

A not so trivial problem...



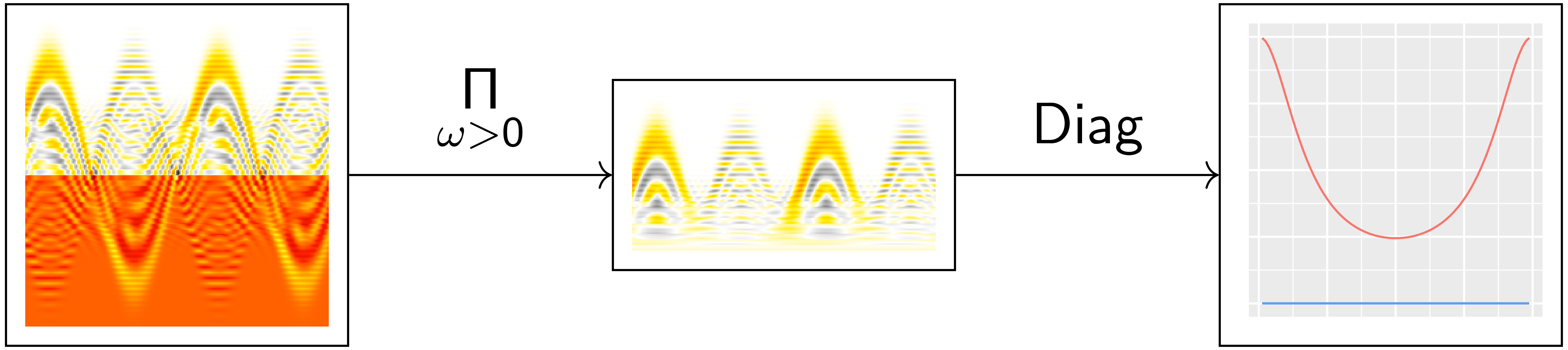


B. Roussel, PhD thesis (tel-01730943)

Extracting Floquet-Bloch waves

ENS Lyon

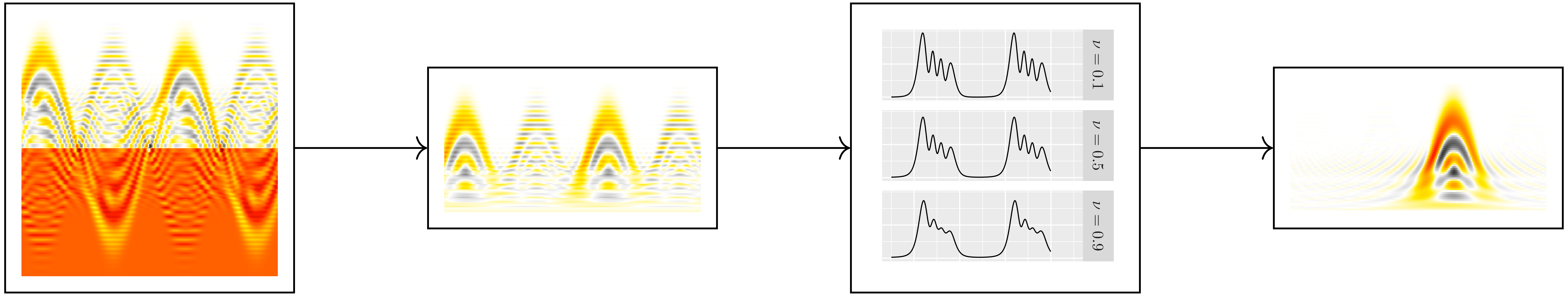
Full coherence (theory/experiment) | **Electronic part of the coherence** | **Spectrum** and eigenmodes



Bloch theory		Electron quantum optics	
a	cell size	T	period
k	quasi-momentum	ν	quasi-pulsation
$\|\psi_n(k)\rangle$	Bloch waves	$\|\psi_n(\nu)\rangle$	eigenmodes
$E_n(k)$	energy spectrum	$p_n(\nu)$	probability spectrum

Extracting « electronic atoms of signal »

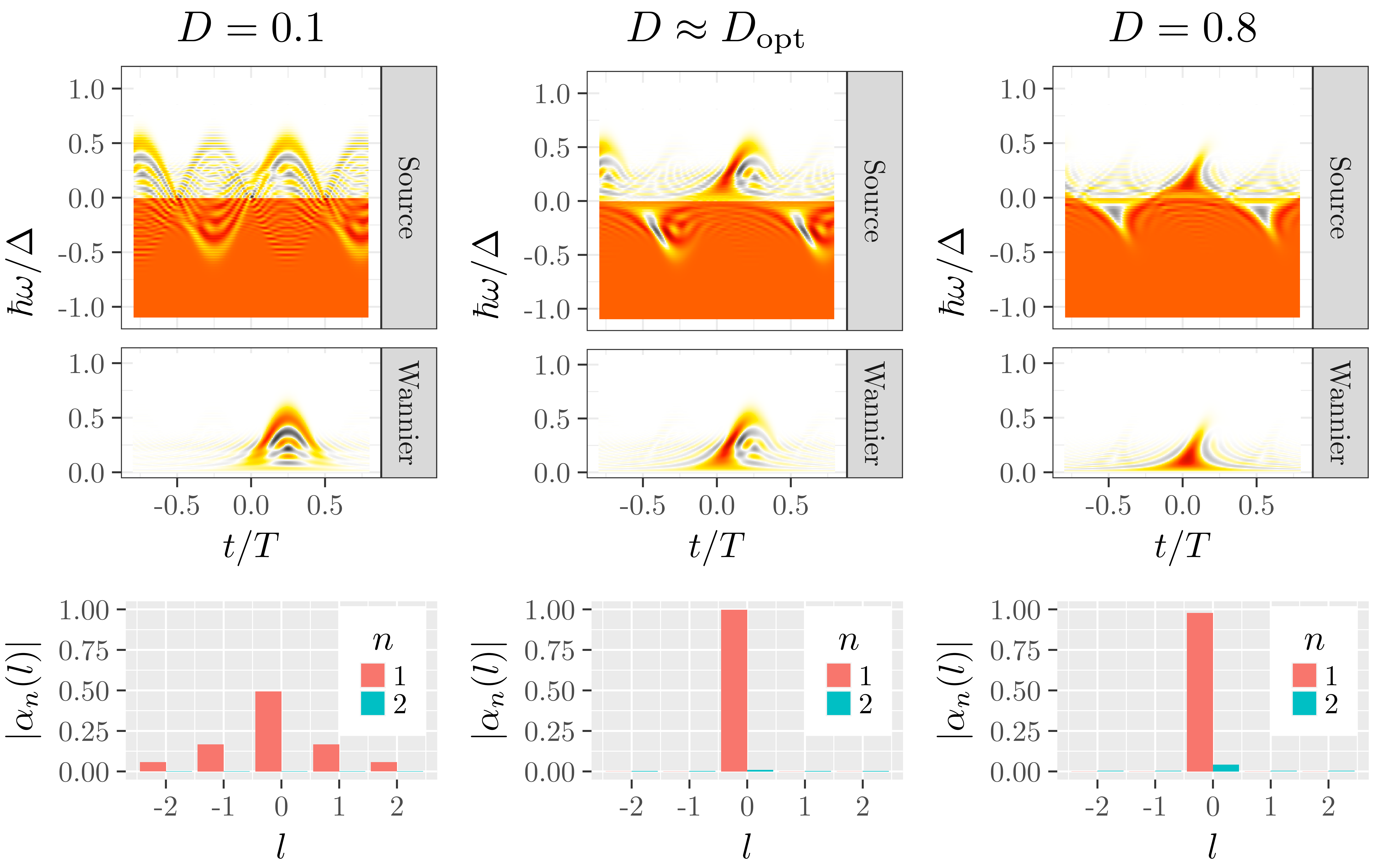
Full coherence (th/exp) → **Electronic part of the coherence** → Spectrum and **eigenmodes** → **Wannier functions**



Basis analoguous to Wannier functions:

- For each band of the spectrum, time-translated Wannier functions
- Coherences from one period to the other in the same band ($\alpha_n(l)$)

Accessing coherences between individual wave packets



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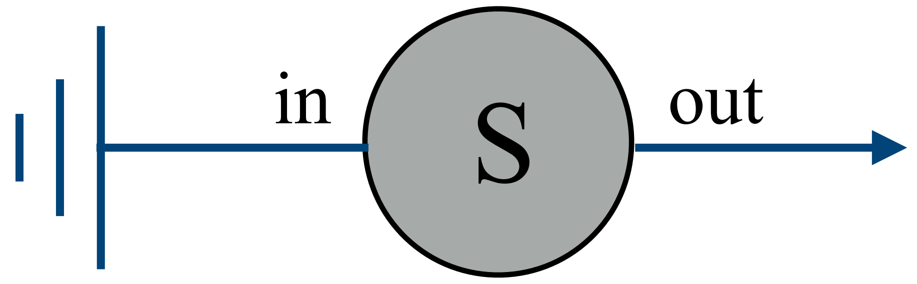
Floquet Bloch spectrum = Entanglement spectrum

Floquet scattering

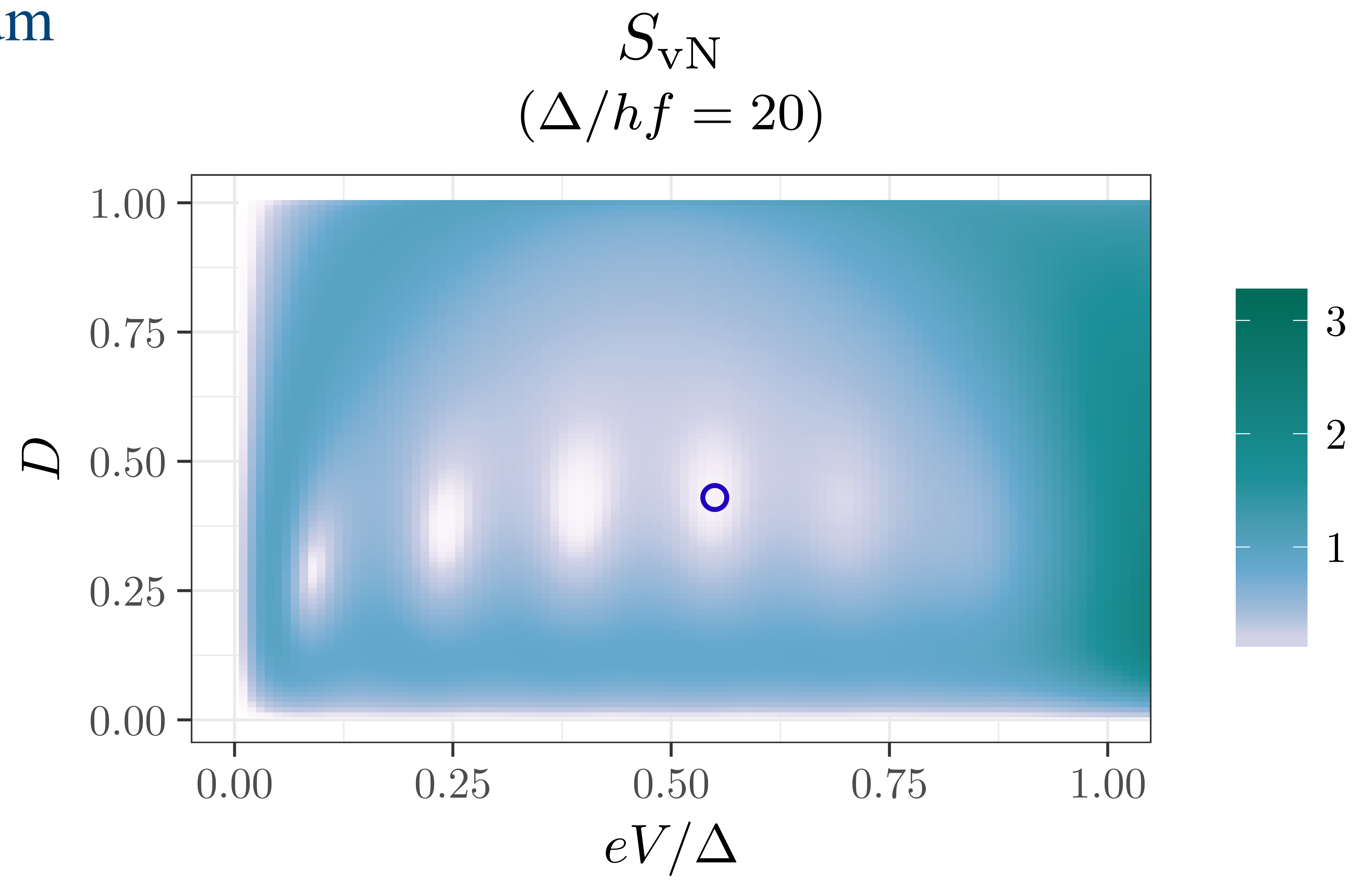


$$\psi_{\text{out}}(t) = \int S(t,t')\psi_{\text{in}}(t')\,dt'$$

Electron/hole entanglement

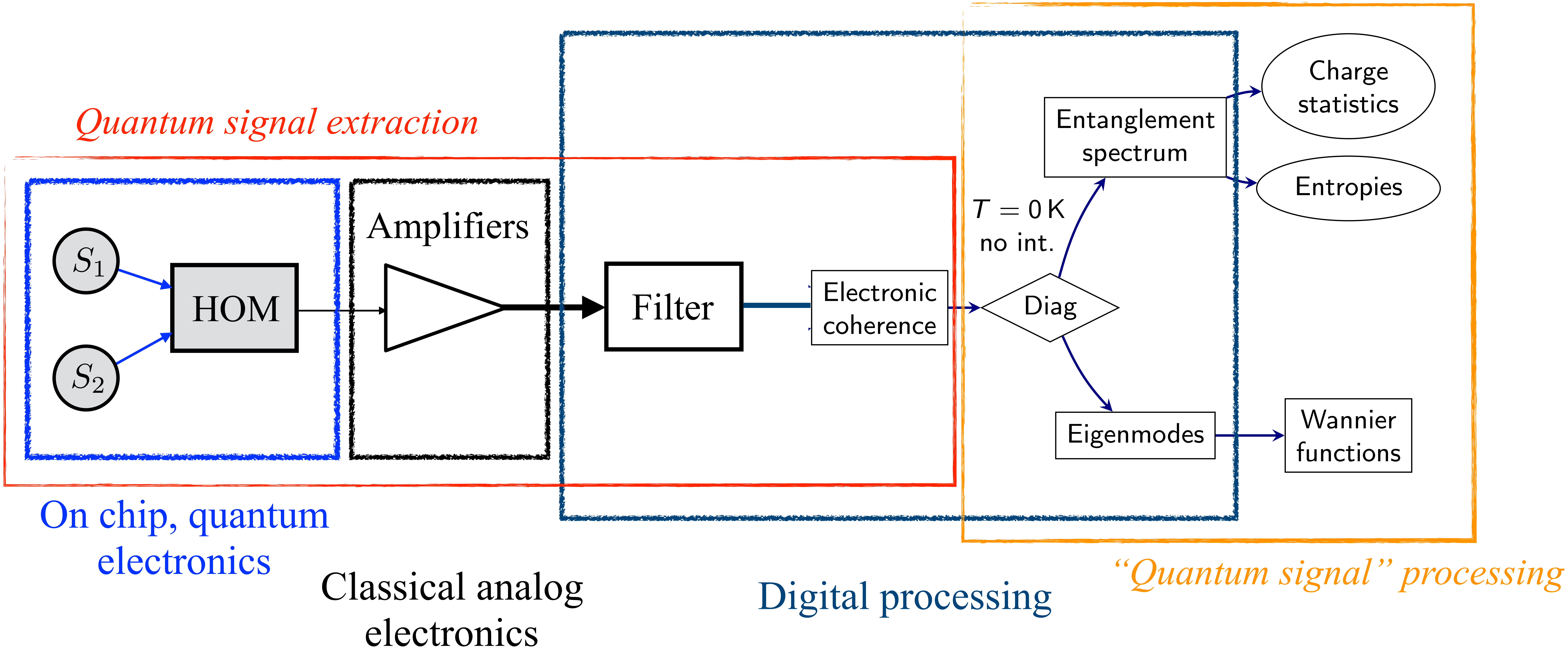
$$|\Psi\rangle = (u + v\,\psi^\dagger[\varphi_e]\psi[\varphi_h])|F_\mu\rangle$$

u and $v \neq 0$



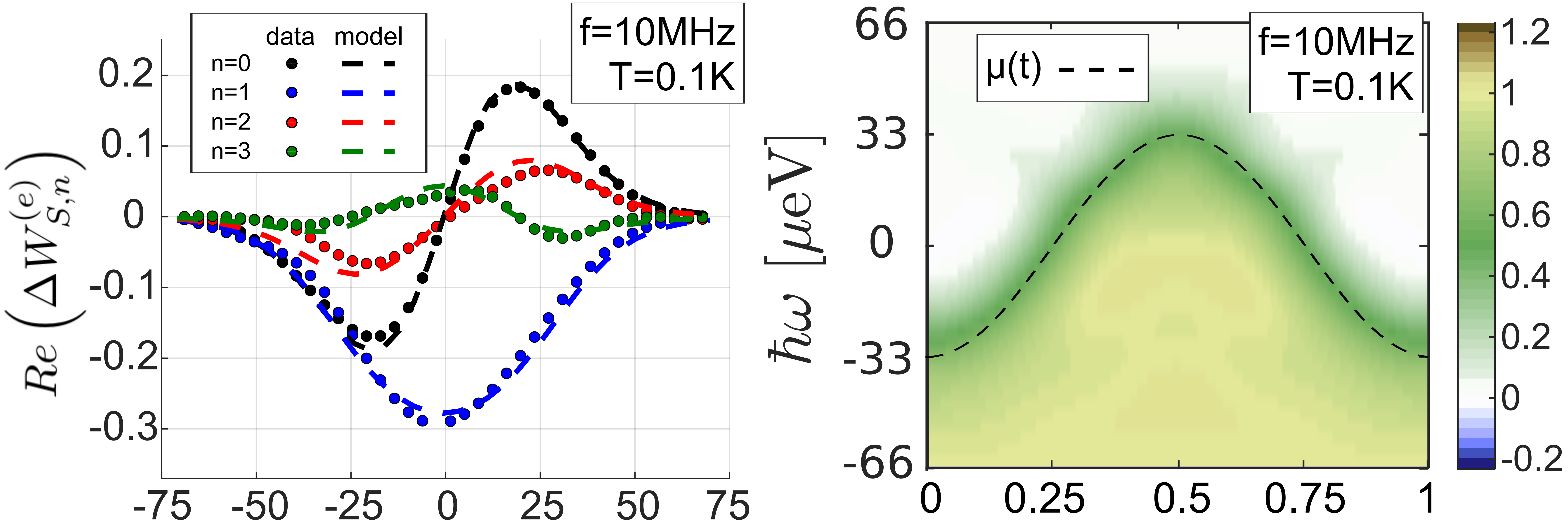
Result: information theoretical measure of e/h entanglement at T=0 K

Autopsy of a quantum electrical current



Experimental results (sinusoidal drive)

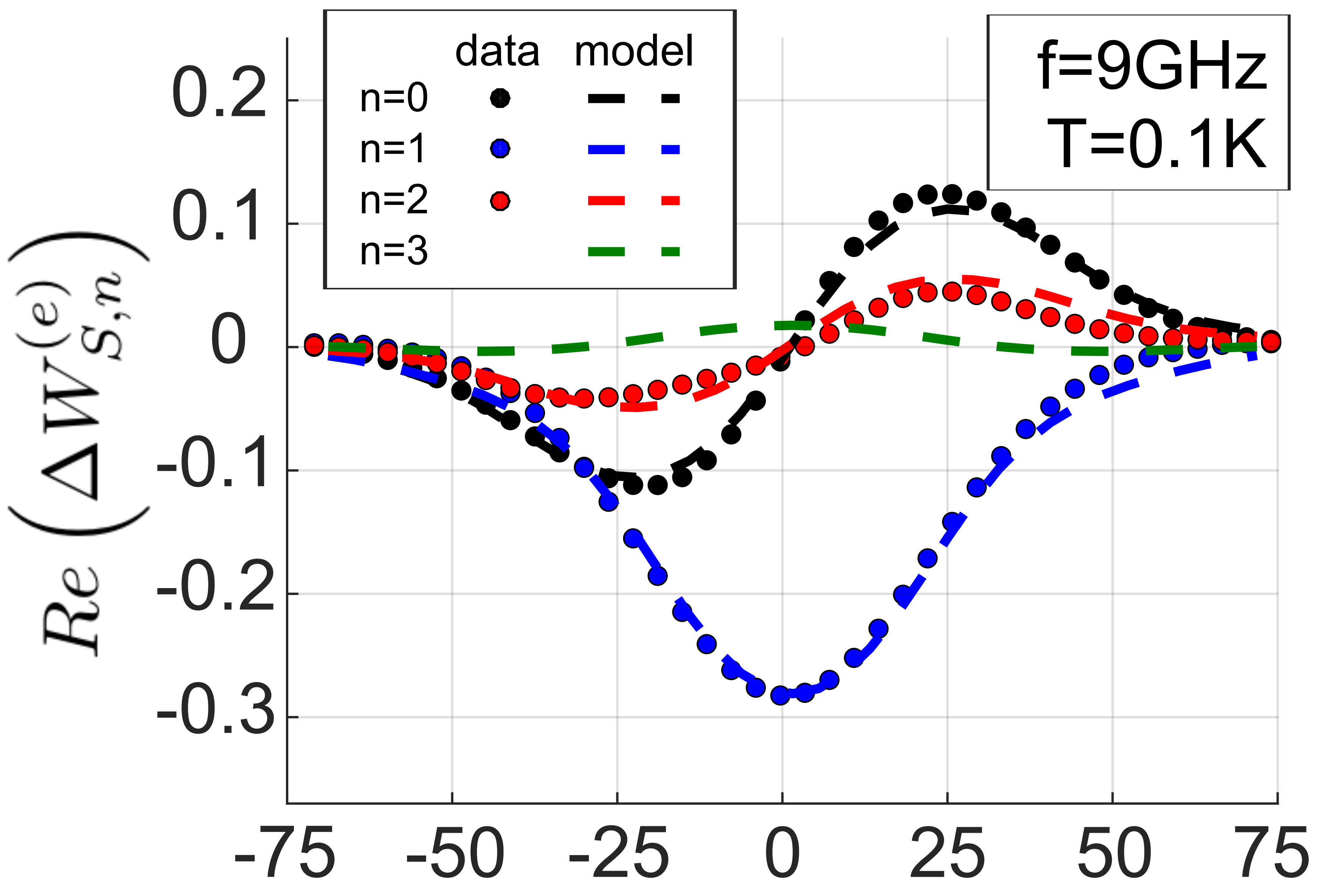
Classical regime : 10 MHz / 100 mK

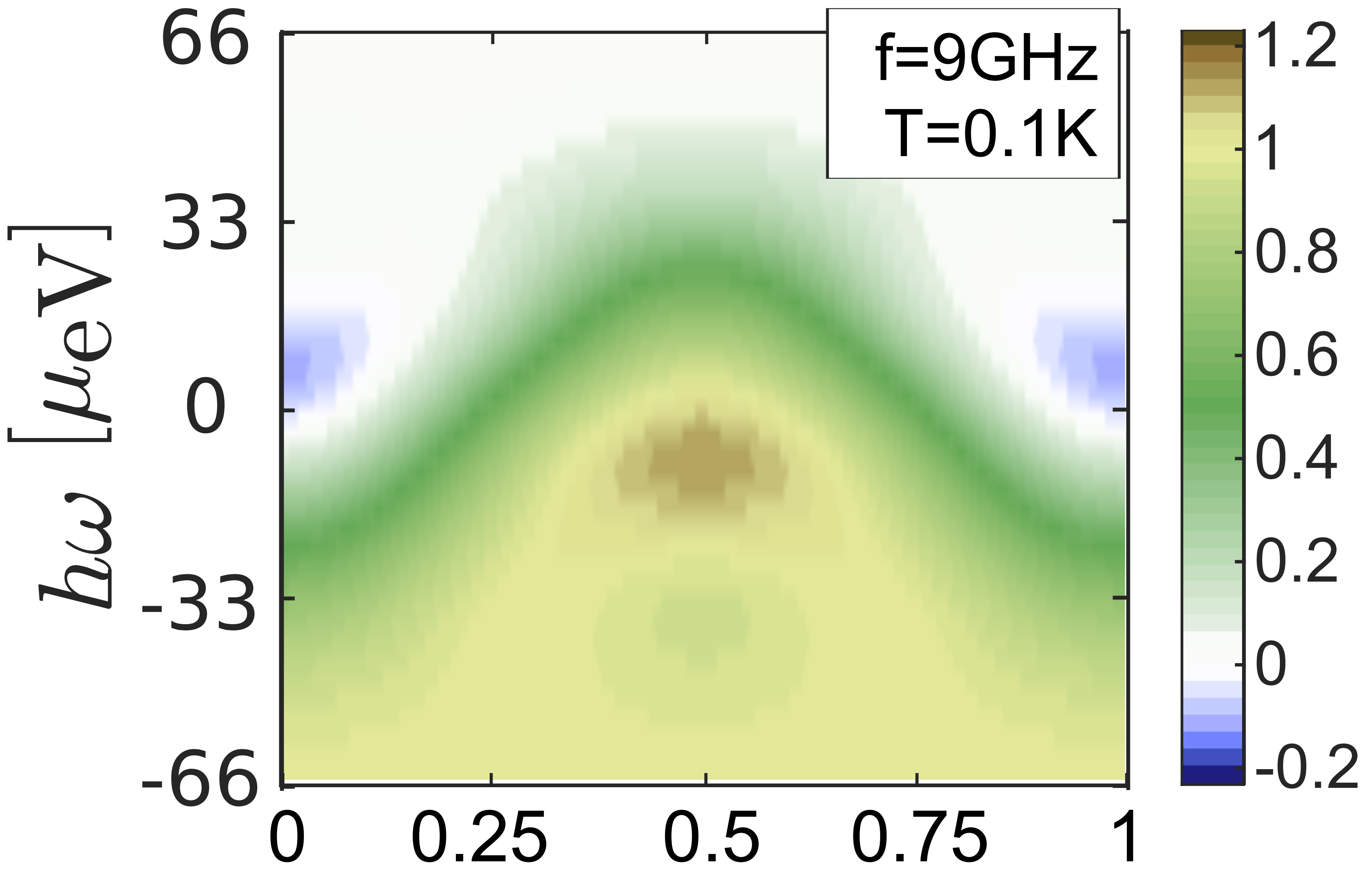


A. Marguerite *et al*, arXiv:1710.11181

Experimental results (sinusoidal drive)

Quantum regime: 9 Ghz / 100 mK





A. Marguerite *et al*, arXiv:1710.11181

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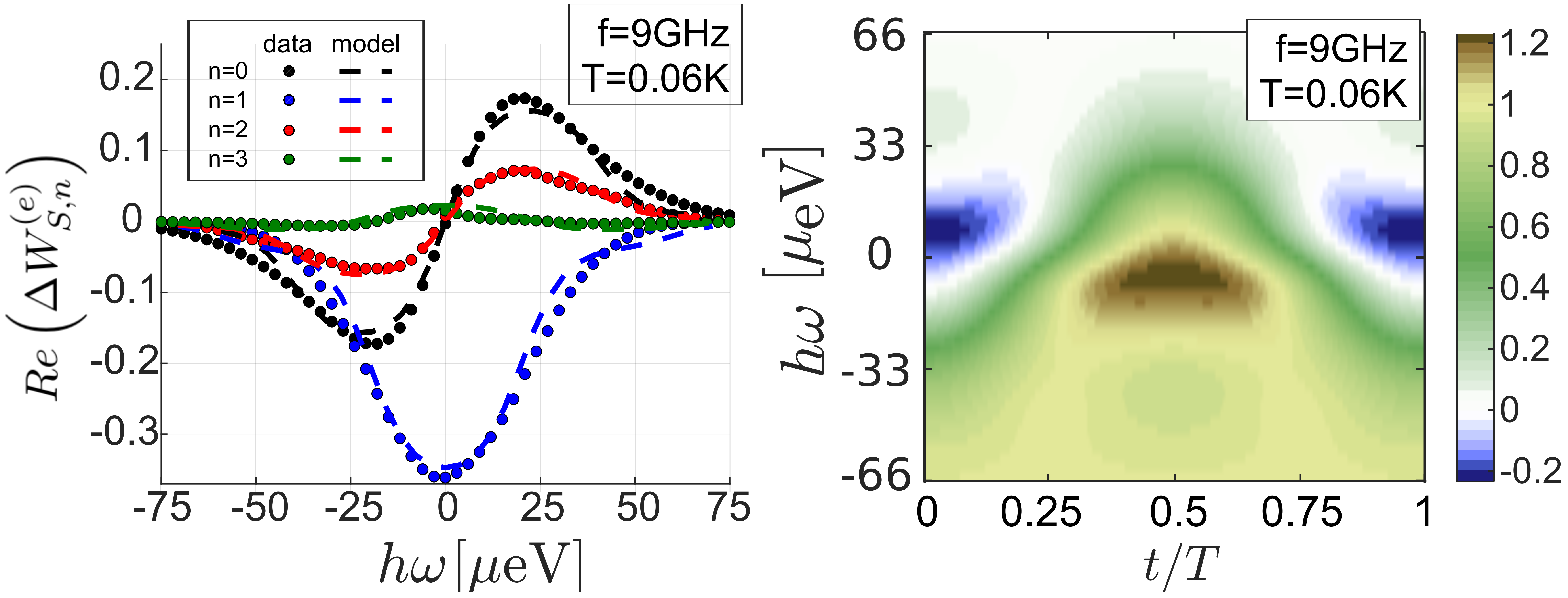
Quantum regime: 9 Ghz / 60 mK



A. Marguerite *et al*, arXiv:1710.11181

Wave function extractions (9 GHz, 60 mK)

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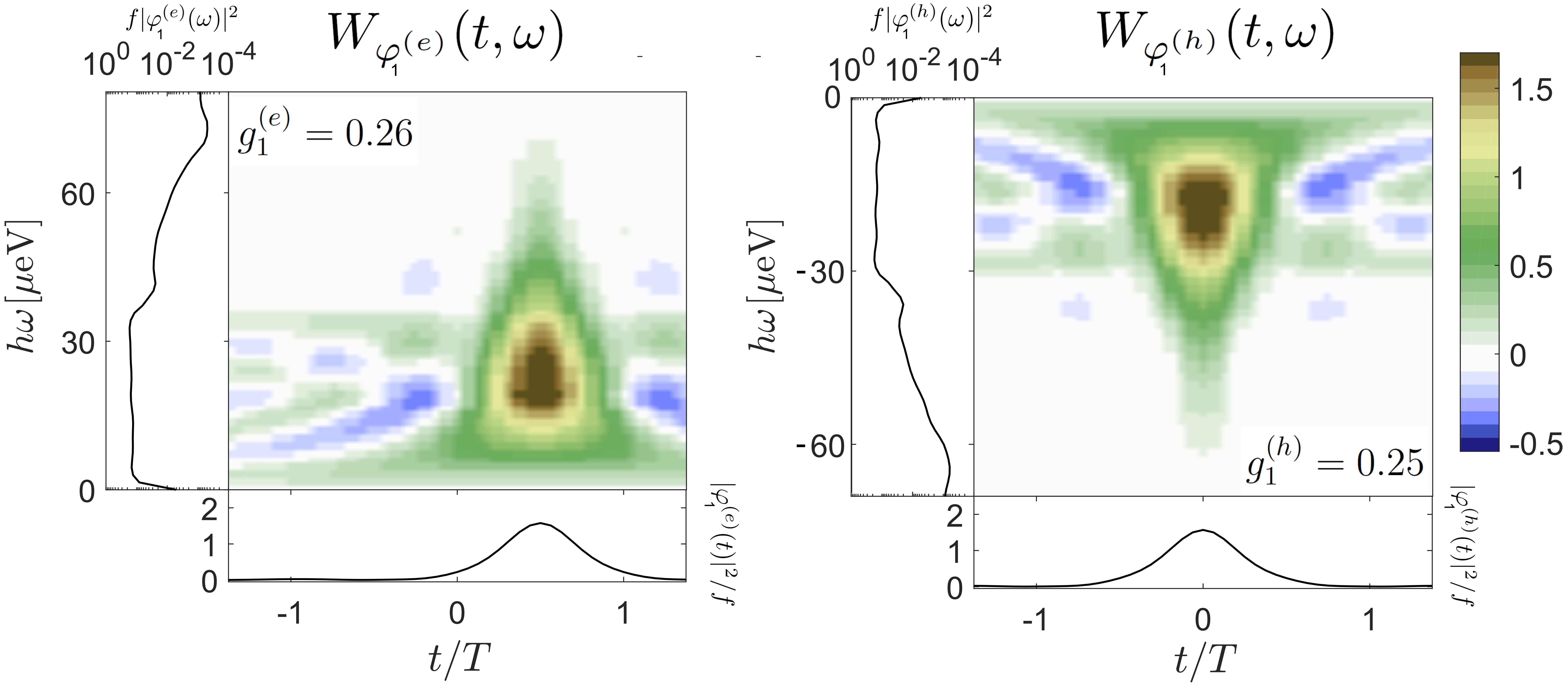
Dominant electron and hole wave functions



A. Marguerite *et al*, arXiv:1710.11181

Electron and hole coherences (sine, 9 GHz, 60 mK)

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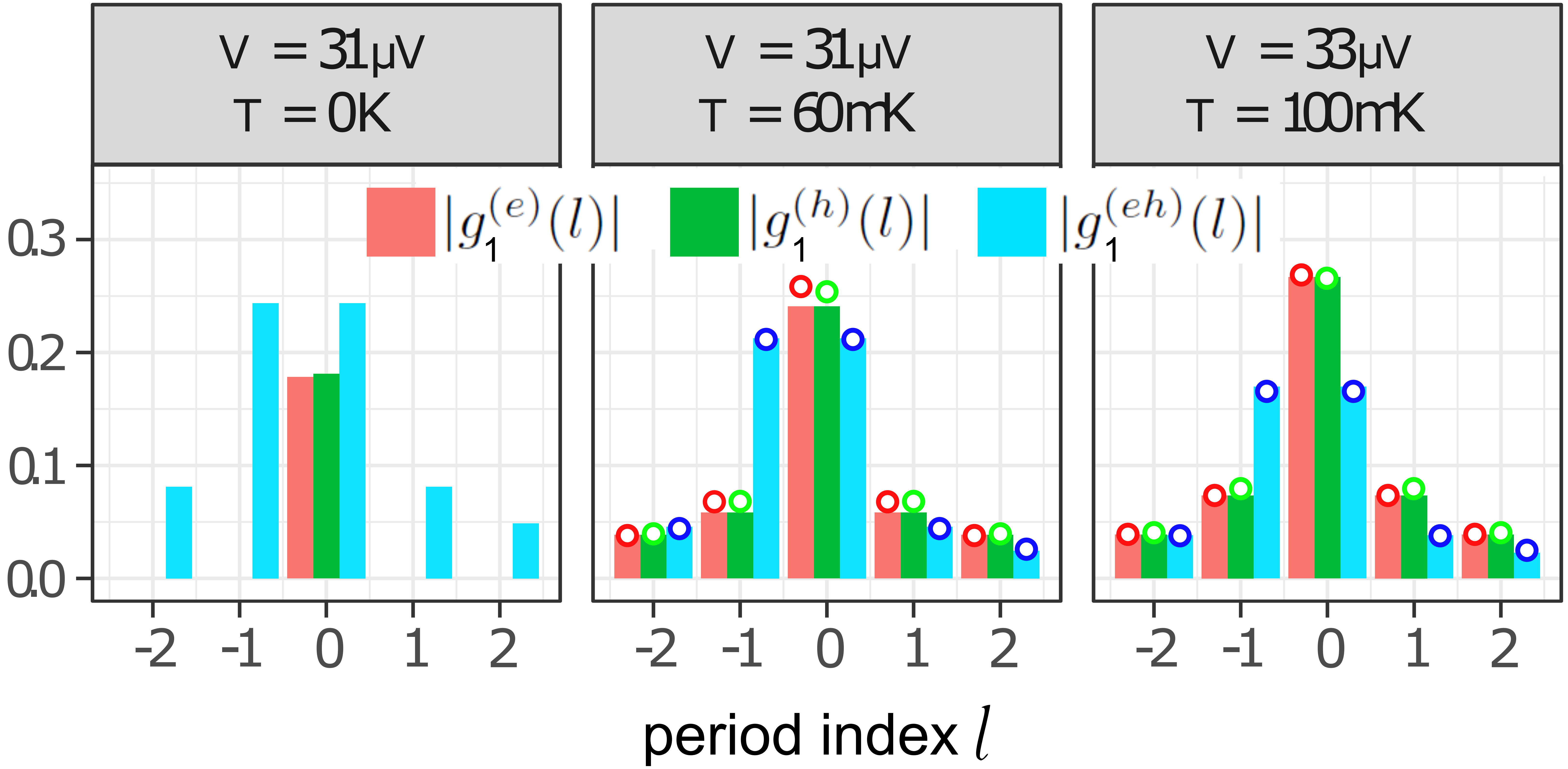
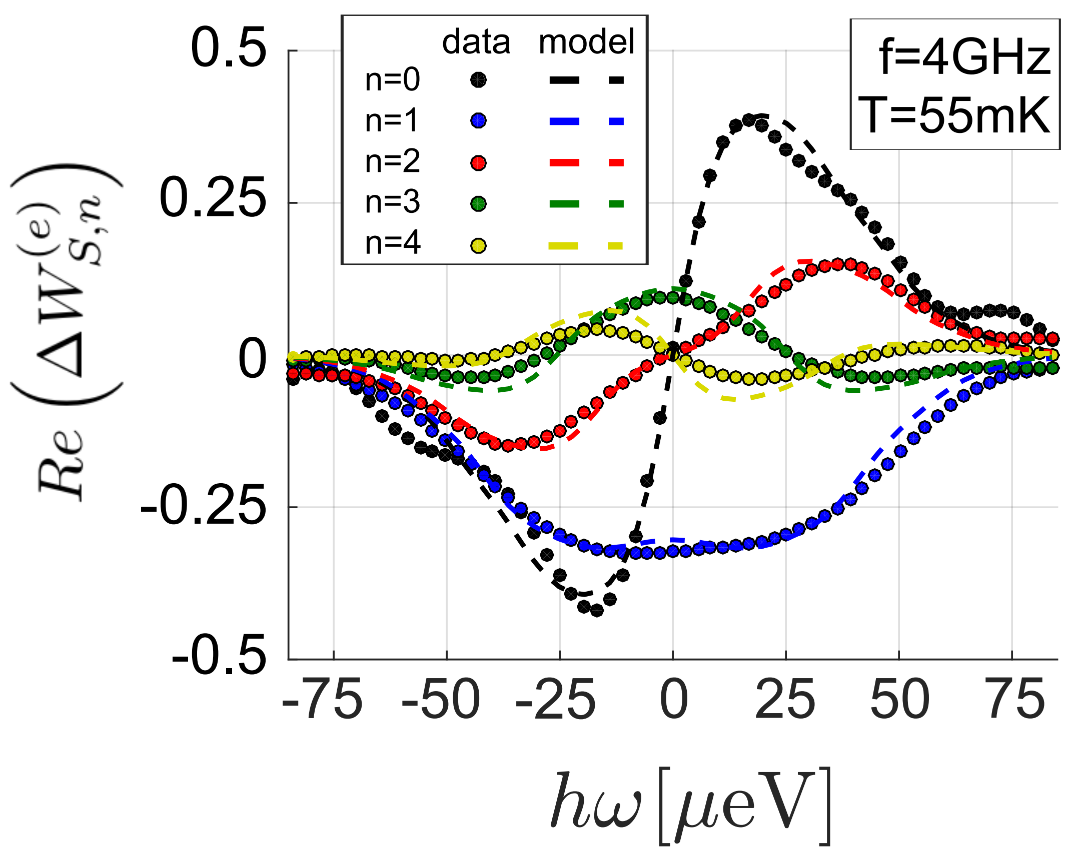


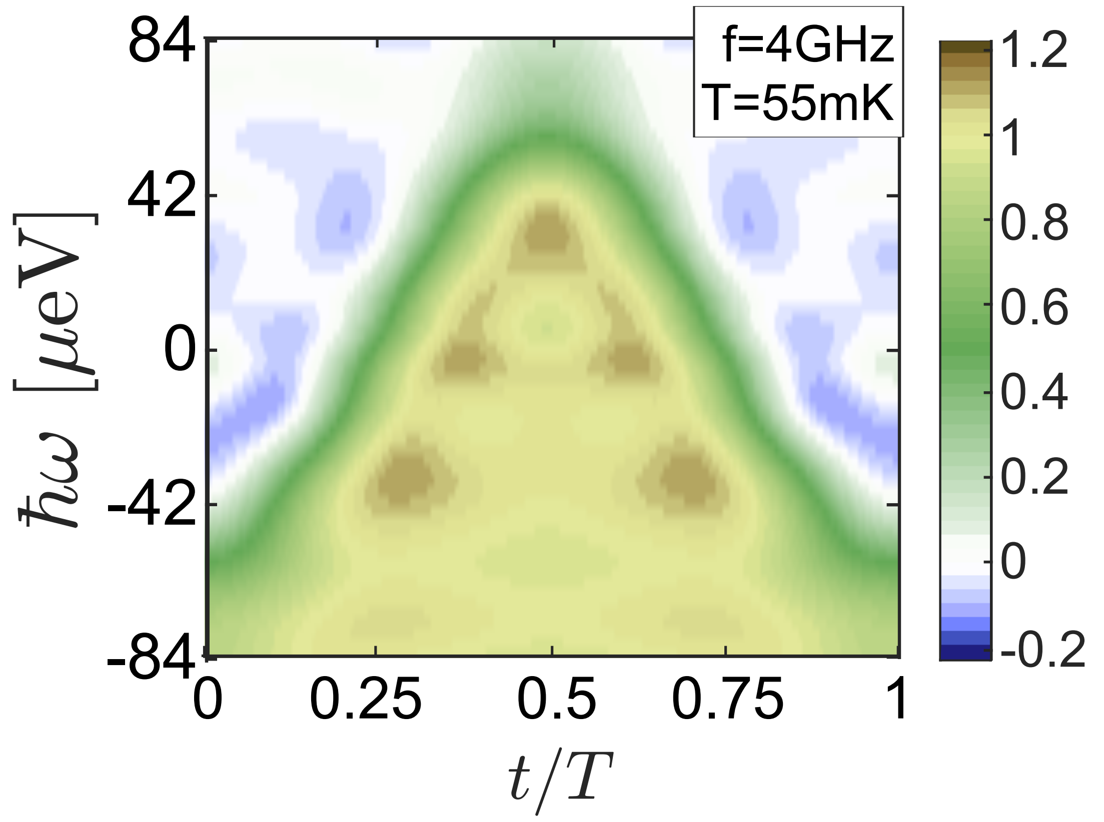
A. Marguerite *et al*, arXiv:1710.11181

What have we done ?

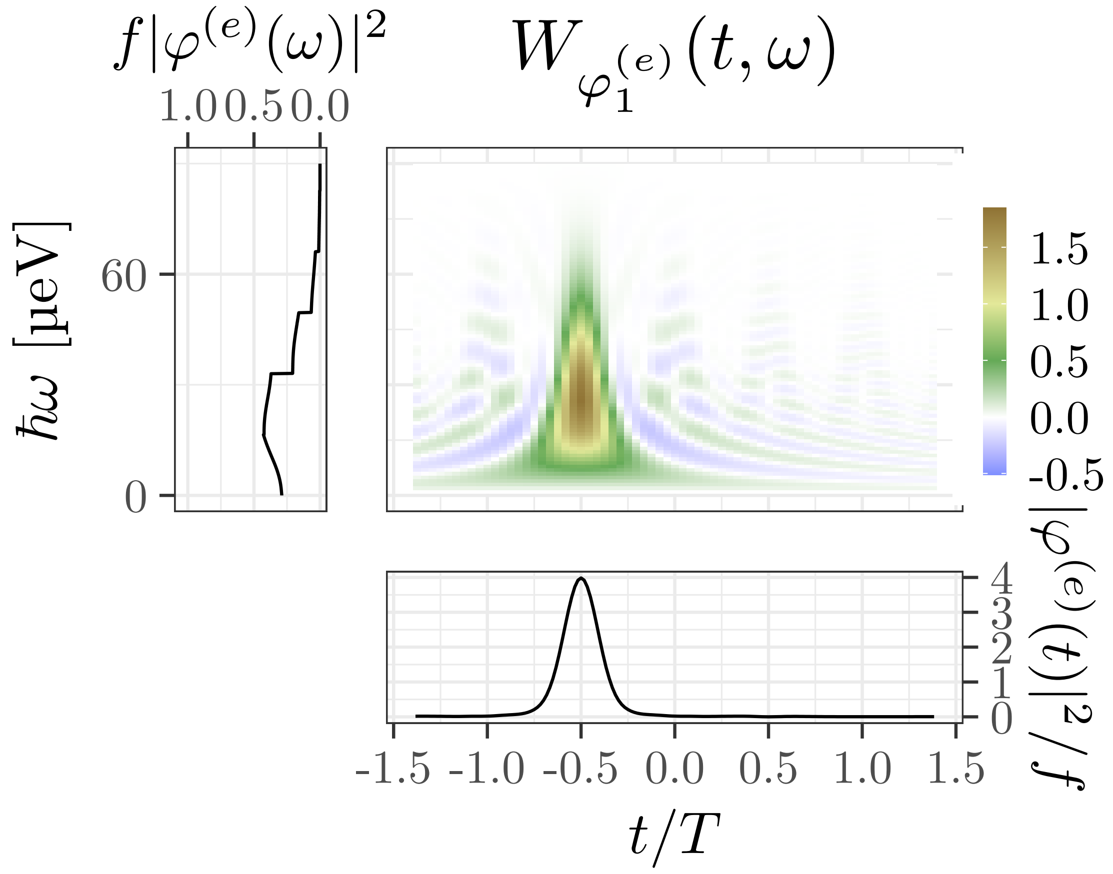
Noise data

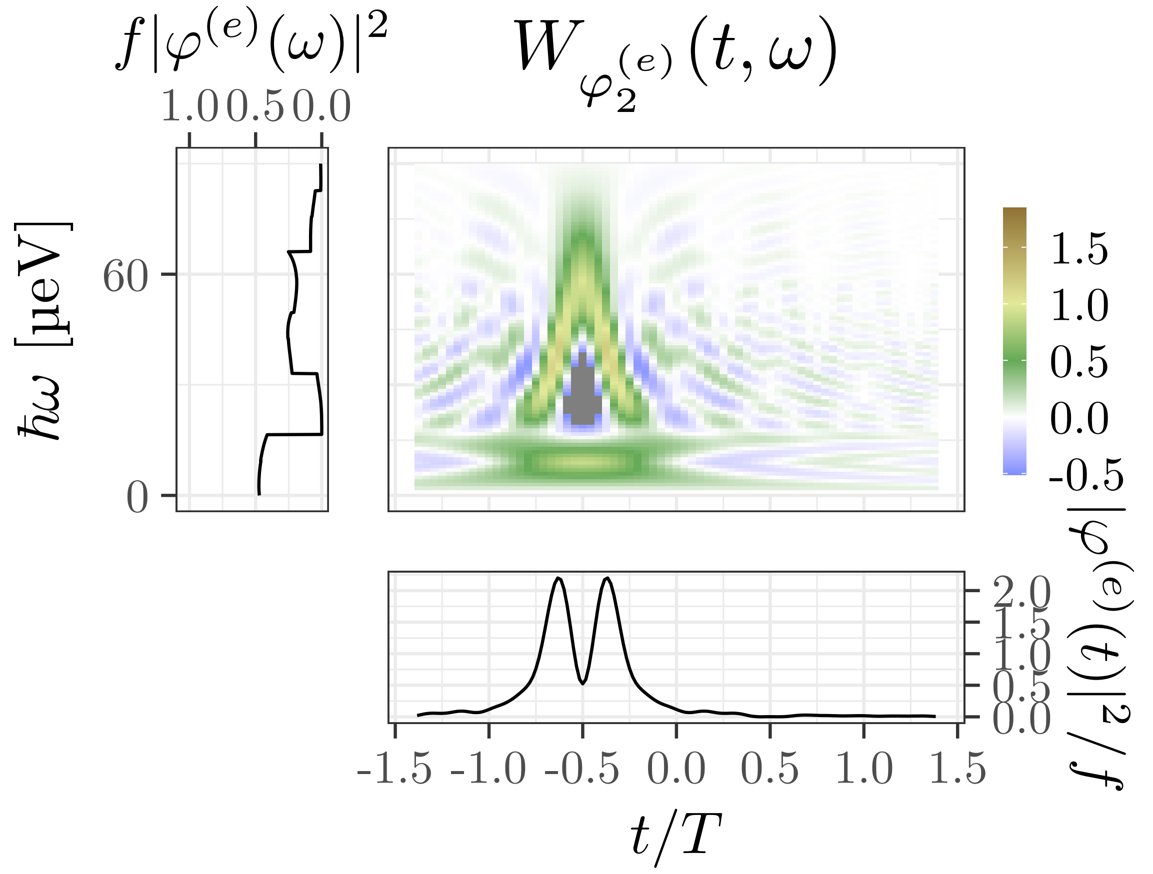


Wigner function



Individual wave functions



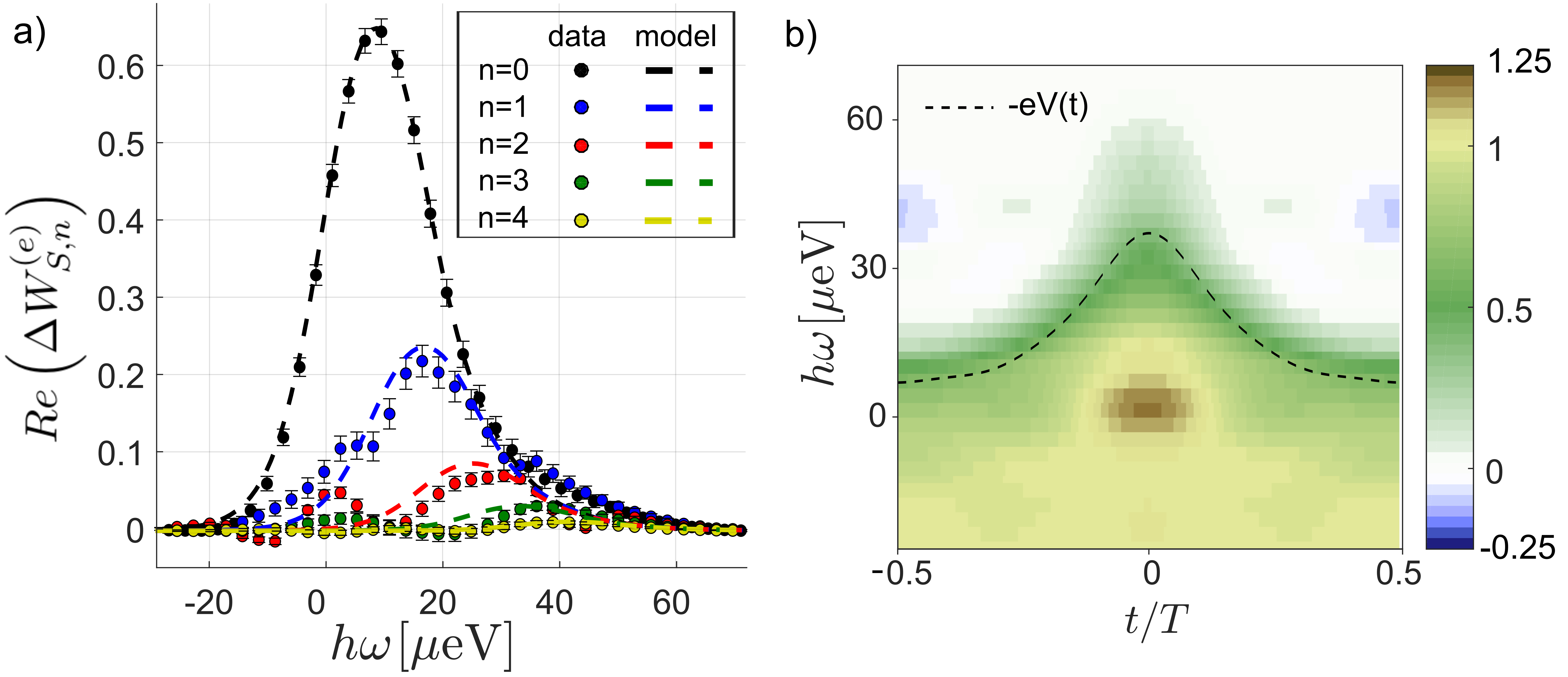


Tomography from HOM interferometry (*aka* « quantum signal processing »)

« Quantum signal » processing

Lorentzian voltage pulses

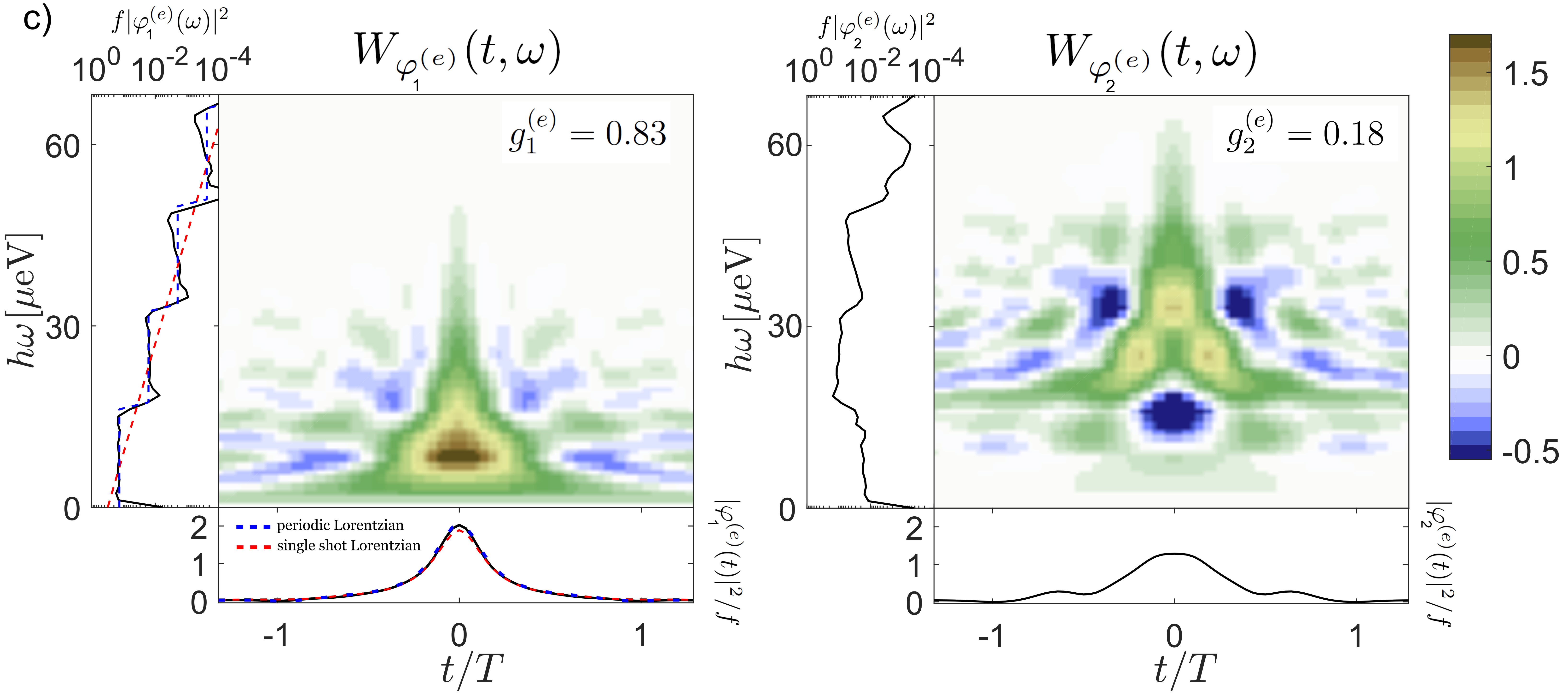
A 40 ps single electron Leviton @50 mK, repeated at 4 GHz



A. Marguerite *et al*, arXiv:1710.11181

Wave function extraction (Lorentzian pulse)

A 40 ps single electron Leviton @50 mK, repeated at 4 GHz



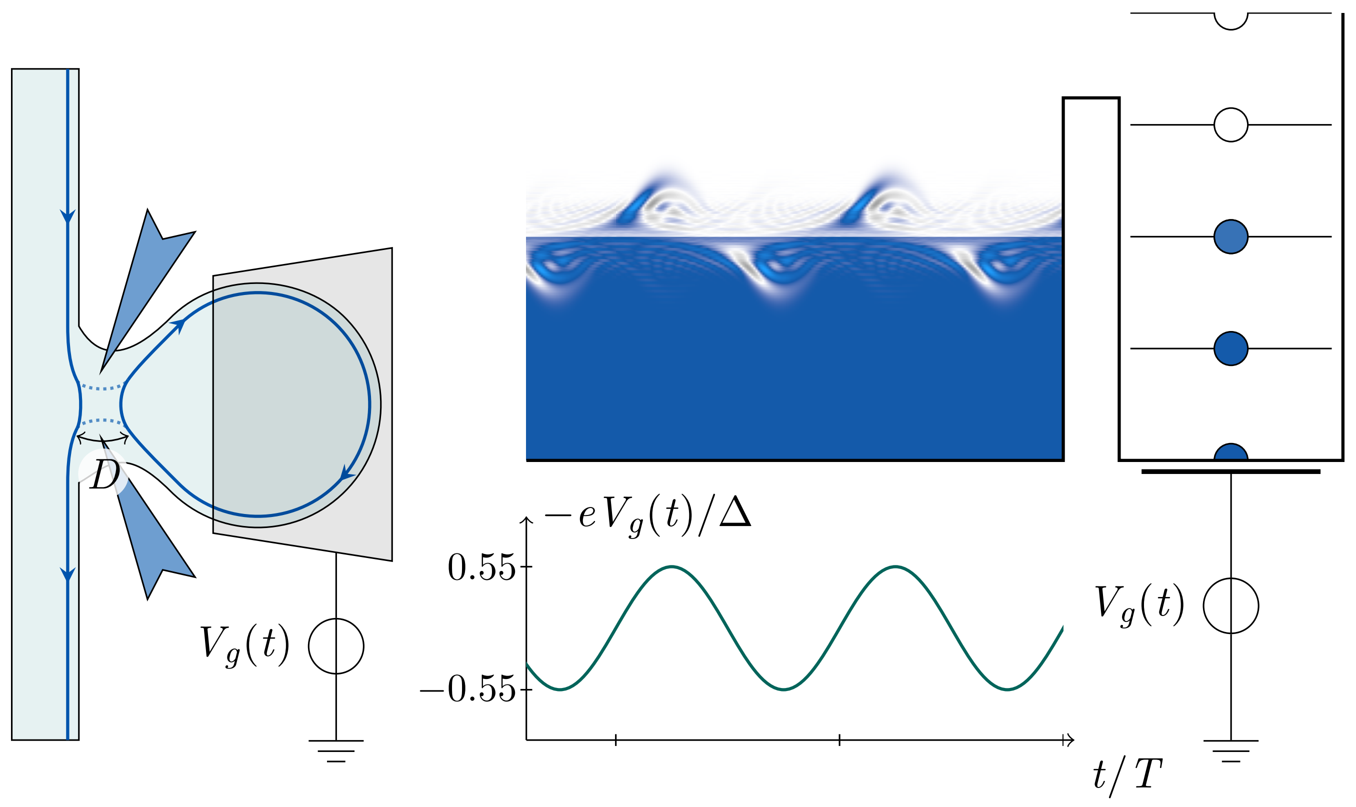
Not really single electronic !!!

A. Marguerite *et al*, arXiv:1710.11181

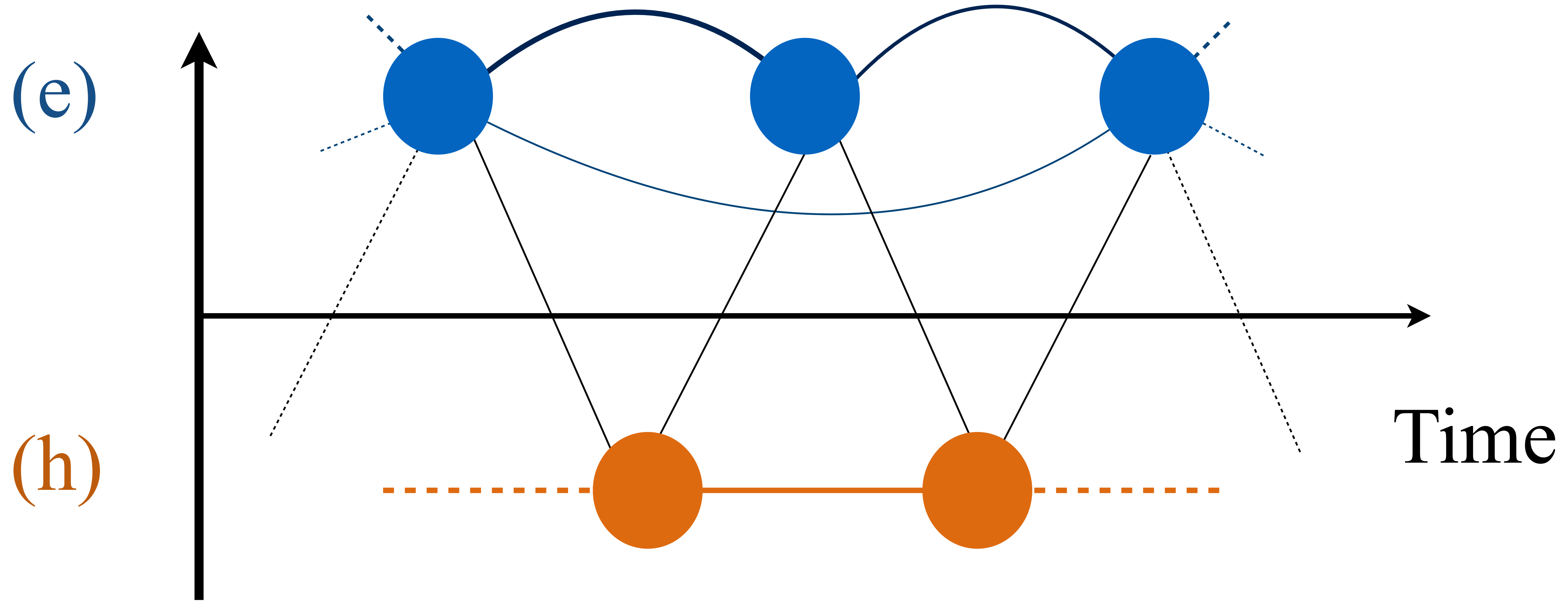
Take home message #3

Single electron coherence can be decomposed into elementary electronic atoms of signal

A proof of concept of the quantum current analyzer has been demonstrated !

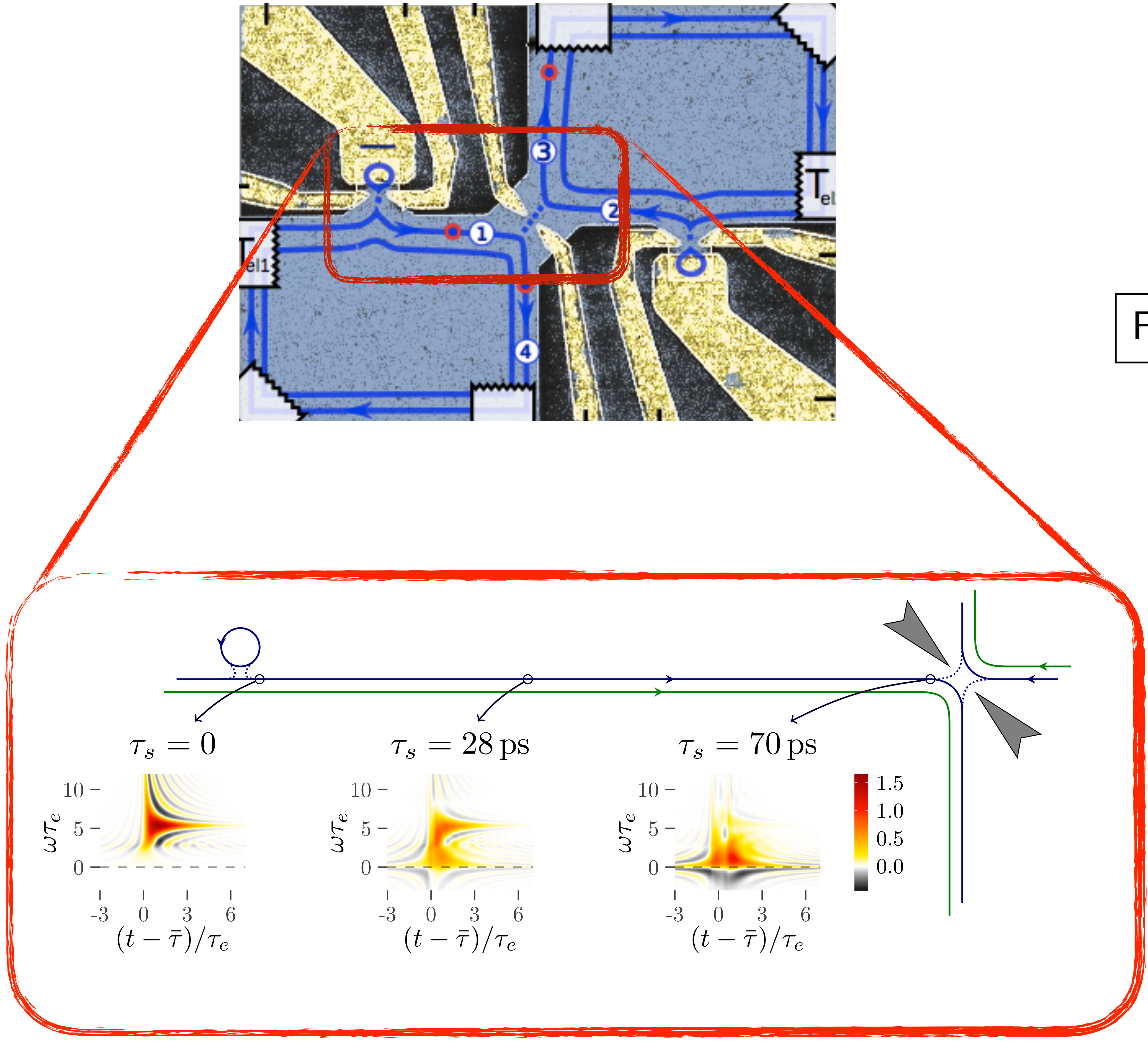


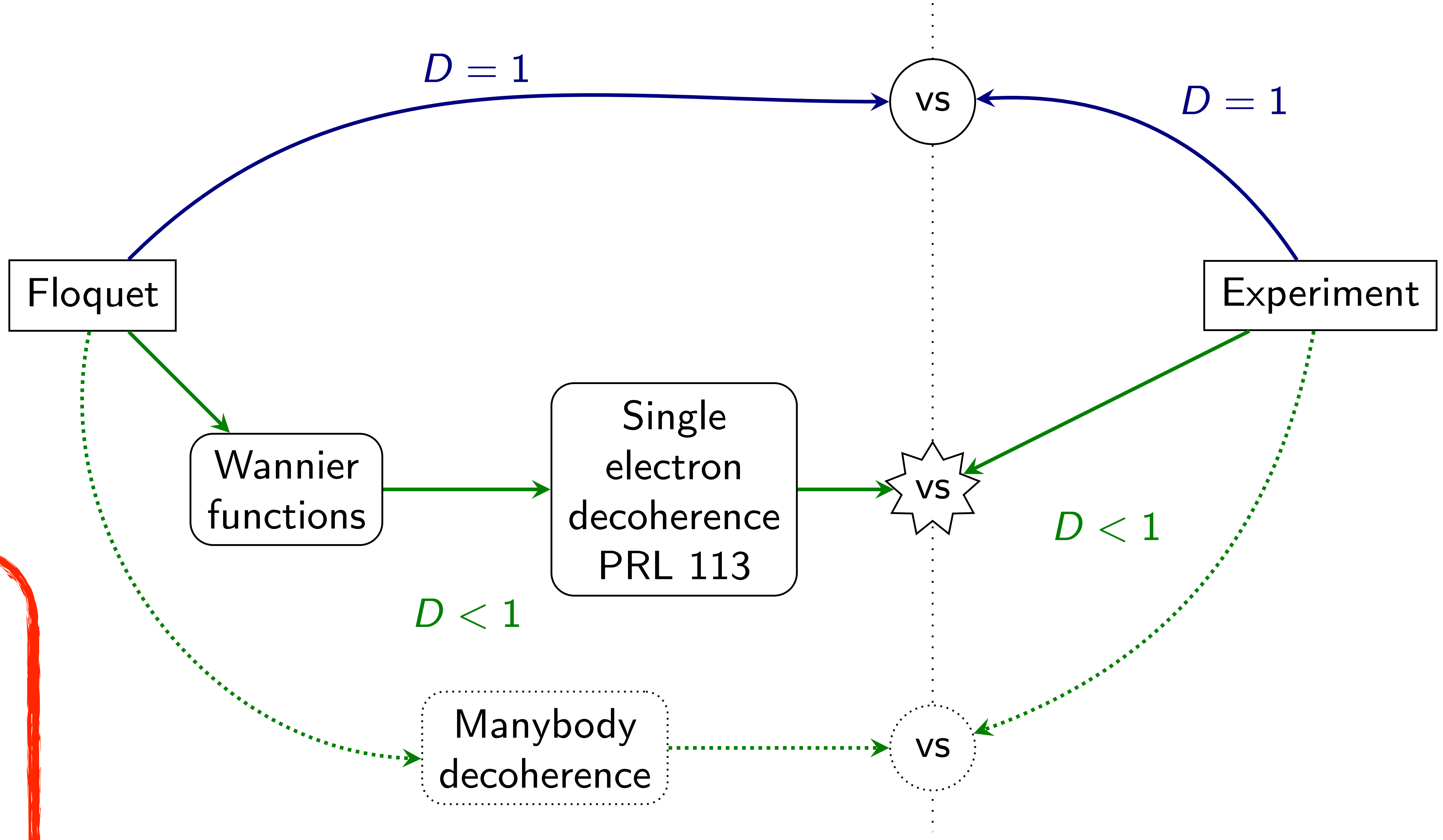
Single electron coherence



« Quantum music »

Theory vs experiment: perspective





Plan

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Analyzing electronic quantum signals: perspetives

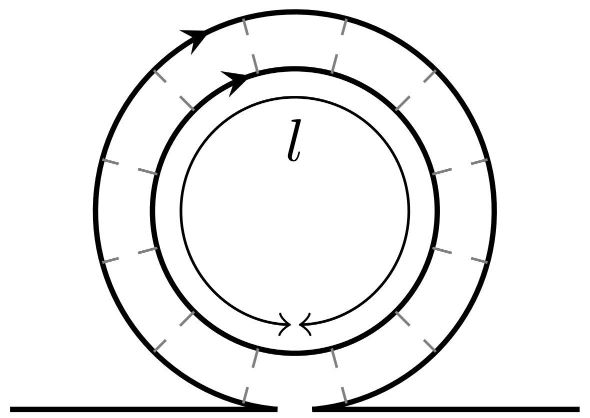
$\Delta\mathcal{G}^{(e)}$

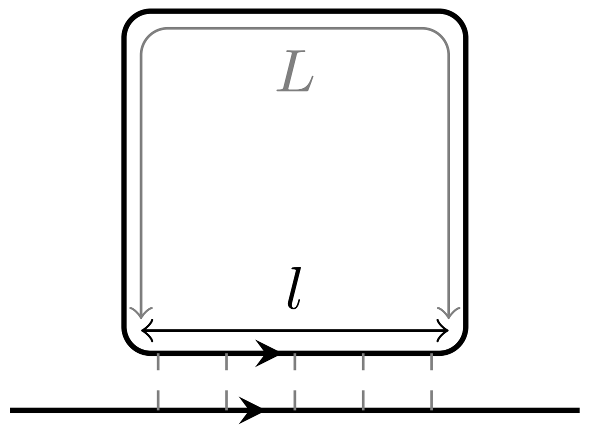
Interferometric measurements (MZI & HOM)

Signal processing of electronic coherence

Single particle physics & electronic decoherence

Decoherence control





$\Delta\mathcal{G}^{(2e)}$

Interferometric measurements (Franson & Samuelsson-Büttiker ?)

Two particle physics: entanglement, interaction induced quantum correlations *etc*

C. Cabart (PhD thesis)

Potential applications : quantum sensing of electric and magnetic fields at the submicron scale.

Two-electron coherence

ENS Lyon

2e-coherence: $\mathcal{G}_{\rho}^{(2e)}(1,2|1',2') = \mathrm{Tr}\left(\psi(2)\psi(1)\rho\,\psi^\dagger(1')\psi^\dagger(2')\right)$ M. Moskalets, Phys. Rev. B. **89**, 045402 (2012)

- Encodes two-electron wave-functions
- Symmetries in 4D space: quantum statistics

$$\prod_{k=1}^{N}\psi^\dagger[\varphi_k]\,|\emptyset\rangle \text{ with } \langle\varphi_k|\varphi_l\rangle = \delta_{k,l}$$

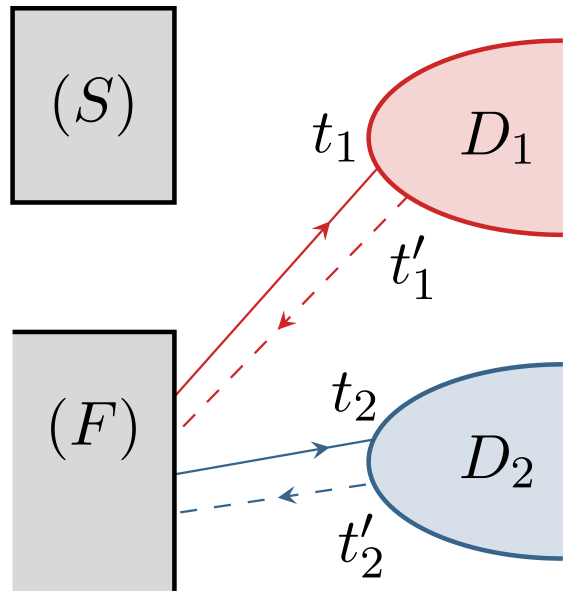
$$\mathcal{G}^{(2e)}(1,2|1',2') = \sum_{\{k,l\}}\varphi_{k,l}(1,2)\,\varphi_{k,l}^*(1',2')$$

where $\varphi_{k,l}(x,y) = \varphi_k(x)\varphi_l(y) - \varphi_k(y)\varphi_l(x)$

Questions:

- Intrinsic contribution of the source to two-electron coherence?
- How to access the intrinsic two-electron coherence emitted by a source?

Intrinsic two electron coherence



$$\mathcal{G}_{\rho}^{(2e)}(1,2|1',2') = \mathcal{G}_{F}^{(2e)}(1,2|1'2')$$

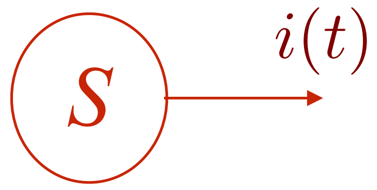
E. Thibierge *et al*, Phys. Rev. B. **93**, 081302(R) (2016)

Accessing two electron coherence

A. Mahé *et al*, Phys. Rev. B **82**, 201309 (2010)
F. Parmentier *et al*, Phys. Rev. B **85**, 165438 (2012)

Current noise measurement

Direct noise measurement:

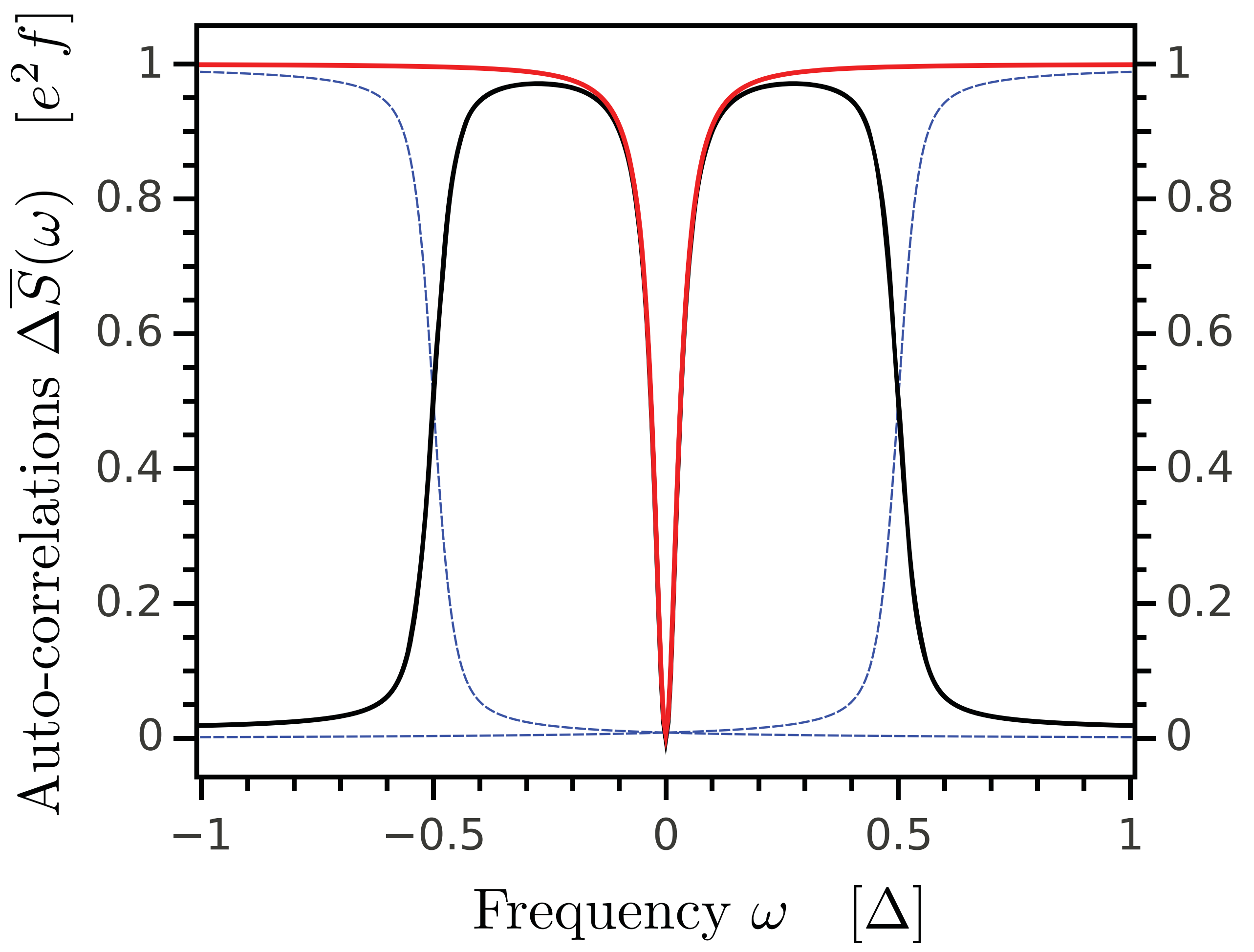


$$S_i(t,t') = \langle i(t)\, i(t')\rangle - \langle i(t)\rangle\langle i(t')\rangle$$

$$\Delta S_i(t,t') = S_i(t,t')_{\mathrm{on}} - S_i(t,t')_{\mathrm{off}}$$

Noise spectrum: $\Delta\overline{S}(\omega) = \int \overline{\Delta S_i(t+\tau/2, t-\tau/2)}^{\,t}\, e^{i\omega\tau}\, \mathrm{d}\tau$

Noise spectrum of the mesoscopic capacitor

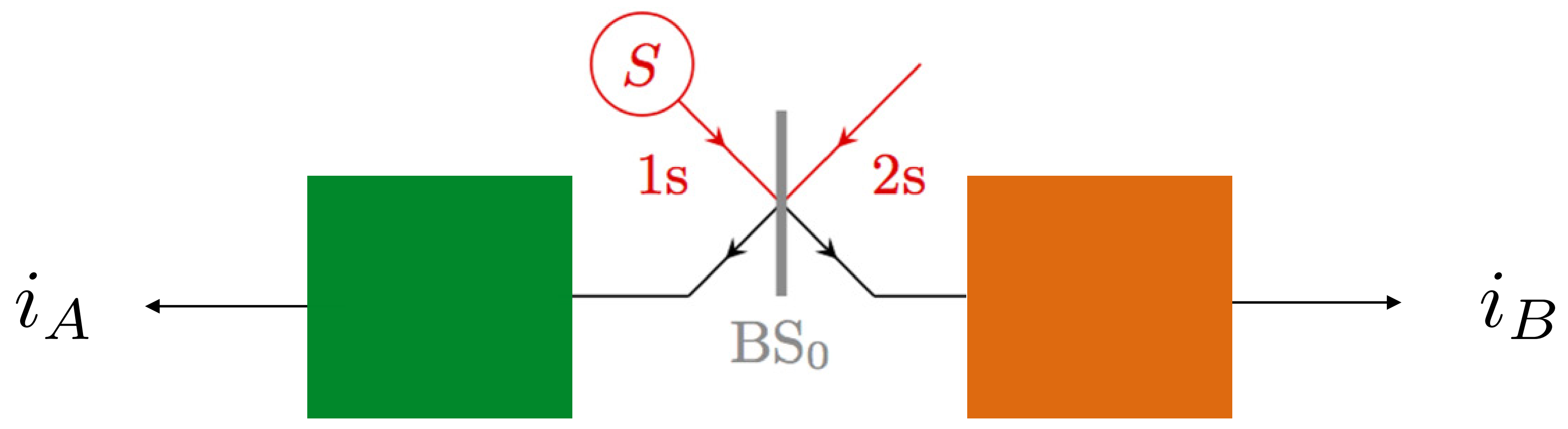


Current noise from electronic coherences

$$\Delta S_i(t,t') = -e\langle i(t)\rangle_S \delta(t-t') + (ev_F)^2 \left(\Delta\mathcal{G}_S^{(2e)}(t,t'|t,t') - \Delta\mathcal{G}_S^{(e)}(t|t)\Delta\mathcal{G}_S^{(e)}(t'|t')\right)$$
$$- (ev_F)^2 \left(\mathcal{G}_F^{(e)}(t|t')\Delta\mathcal{G}_S^{(e)}(t|t') + \mathcal{G}_F^{(e)}(t'|t)\Delta\mathcal{G}_S^{(e)}(t'|t)\right)$$

B. Roussel *et al*, Physica Status Solidi B **254**, 1600621 (2017)

Accessing two electron coherence



A-detector
Signal: outgoing currents

B-detector
Signal: outgoing currents

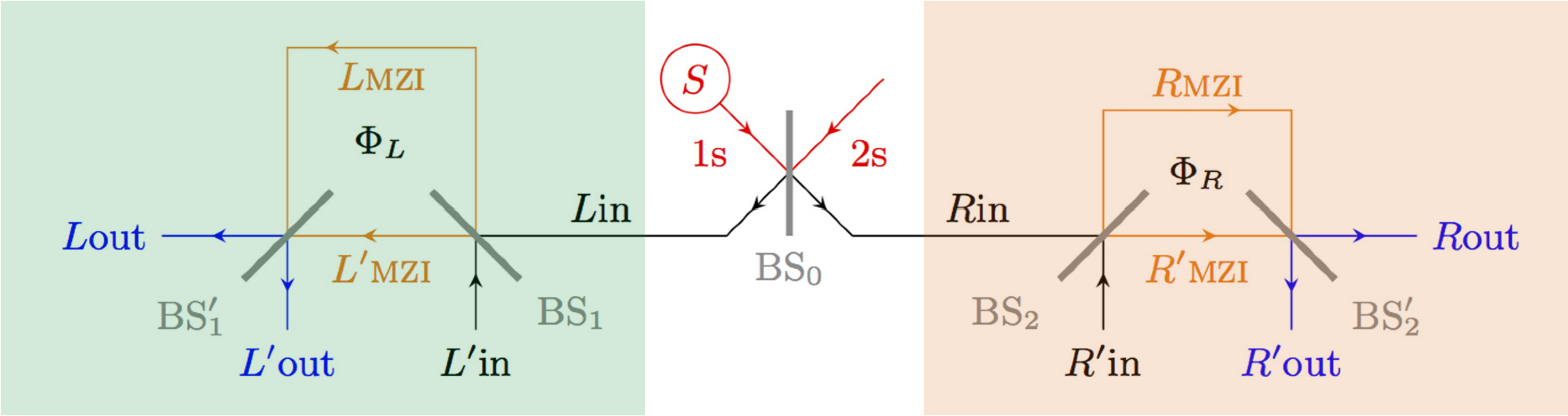
Two particle interferences at the beam splitter: $\Delta\mathcal{G}^{(2e)}_{\mathrm{out}_{\mathrm{BS}_0}}(1\,t_1;2\,t_2|1\,t'_1;2\,t'_2) = RT\,\Delta\mathcal{G}^{(2e)}_{S}(t_1,t_2|t'_1,t'_2)$

Current correlations after the detectors: $\langle i_A\, i_B\rangle = \left(\mathcal{L}^{(1)}_A \otimes \mathcal{L}^{(2)}_B\right)\left[\Delta\mathcal{G}^{(2e)}_{\mathrm{out}_{\mathrm{BS}_0}}(1\,t_1;2\,t_2|1\,t'_1;2\,t'_2)\right]$

It combines:

- HBT interferometry: partitioning of two-electron coherence at a beam splitter
- Single particle interferometry: converting off-diagonal single-electron coherence into measurable signal

Example: Franson interferometry



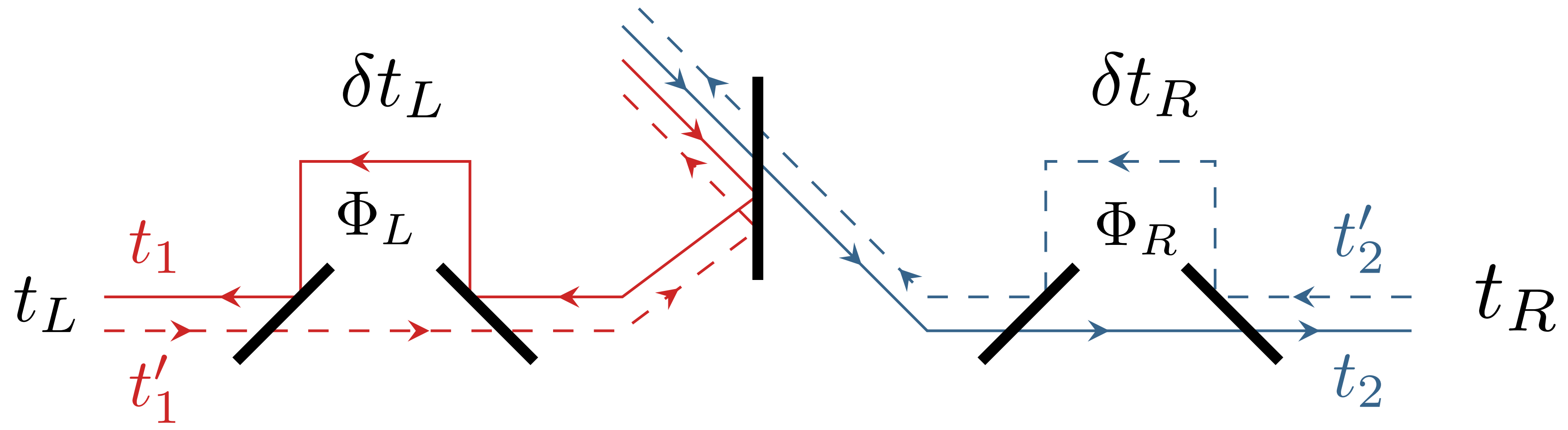
A-detector

Signal: outgoing currents
Parameters: time of flights, **AB flux**

B-detector

Signal: outgoing currents
Parameters: time of flights, **AB flux**

The Franson signal: current correlations between left/right detectors with both flux sensitivities

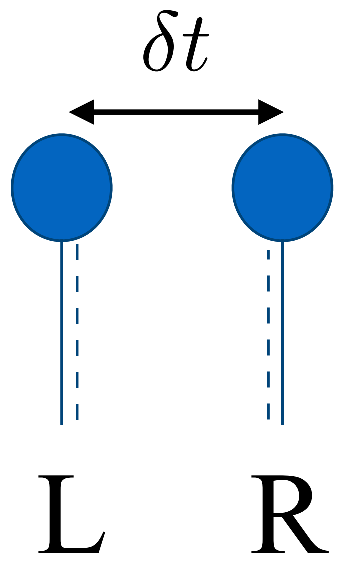


$$\sim (ev_F)^2 e^{-i(\Phi_L+\Phi_R)} \Delta\mathcal{G}_S^{(2e)}(t_L, t_R | t_L - \delta t_L, t_R - \delta t_R)$$

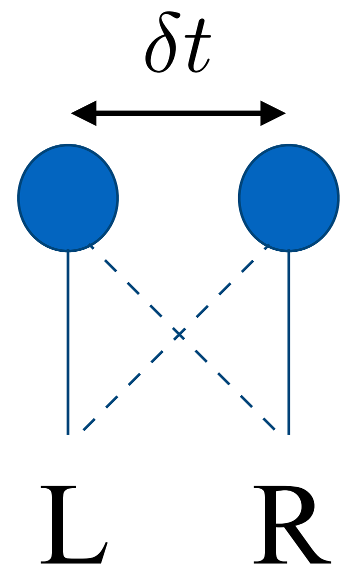
Franson signals

An electron pair

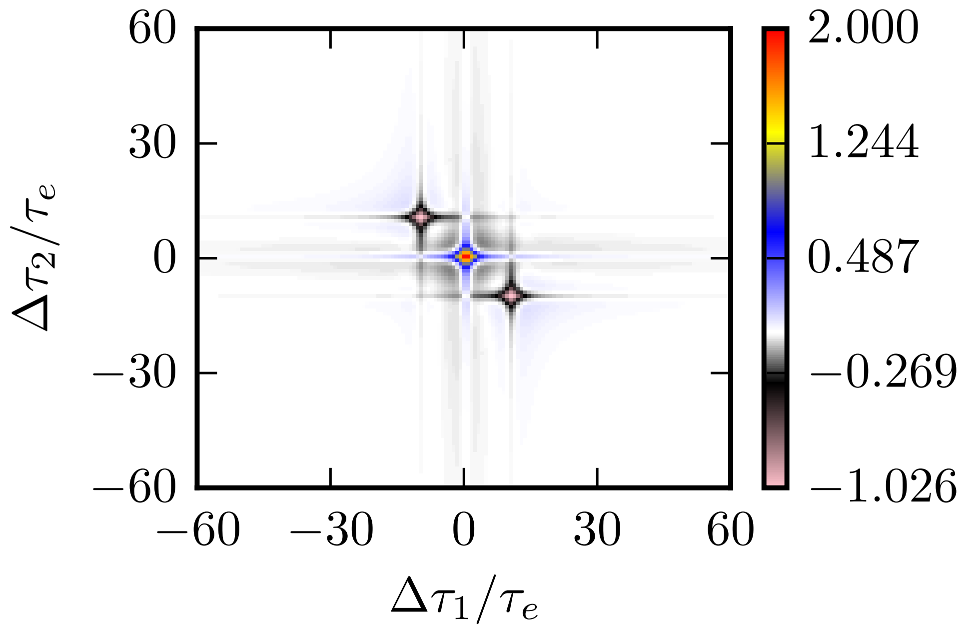
Two Levitons separated by 10x their width τ_0



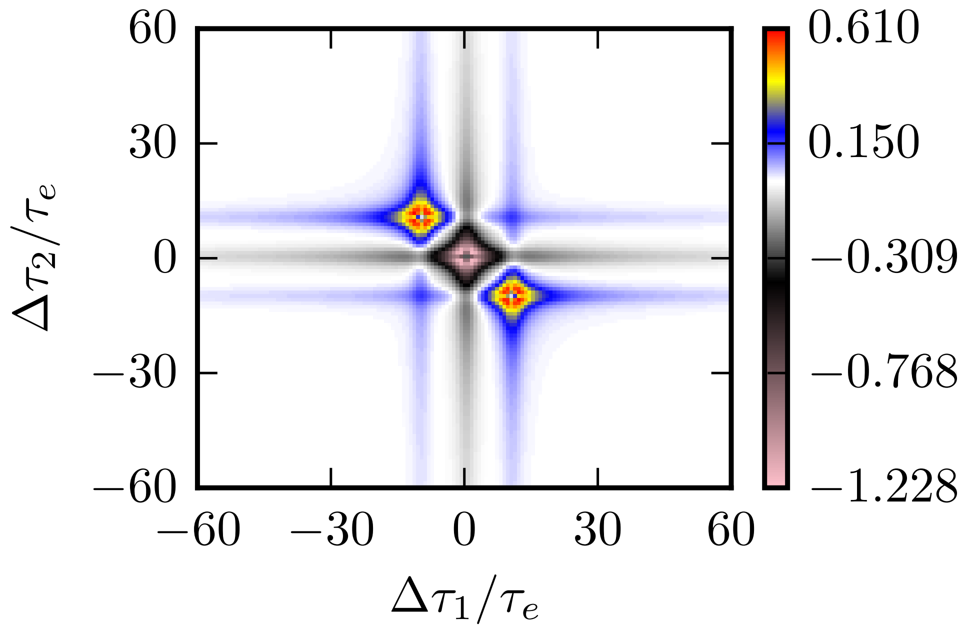
$$\Delta\tau_1 \simeq -\Delta\tau_2 \simeq \tau_0$$



$$\Delta\tau_1 \simeq -\Delta\tau_2 \simeq \delta t$$



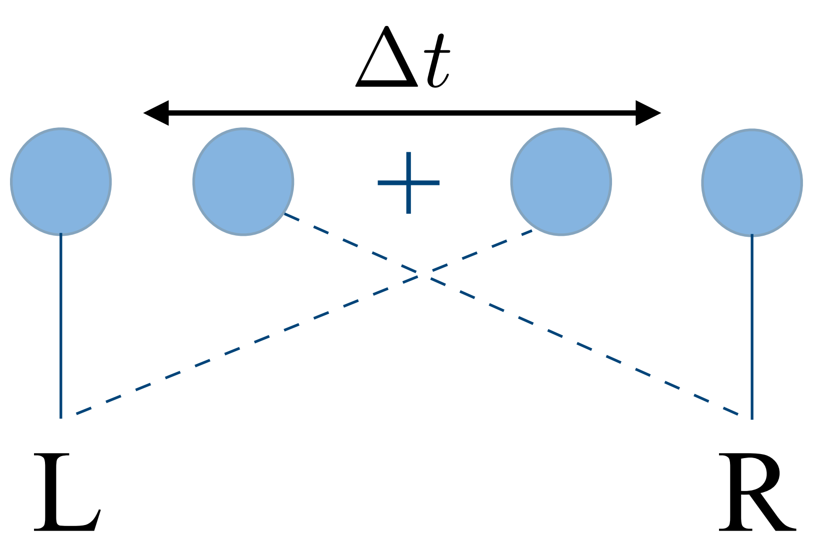
Real part



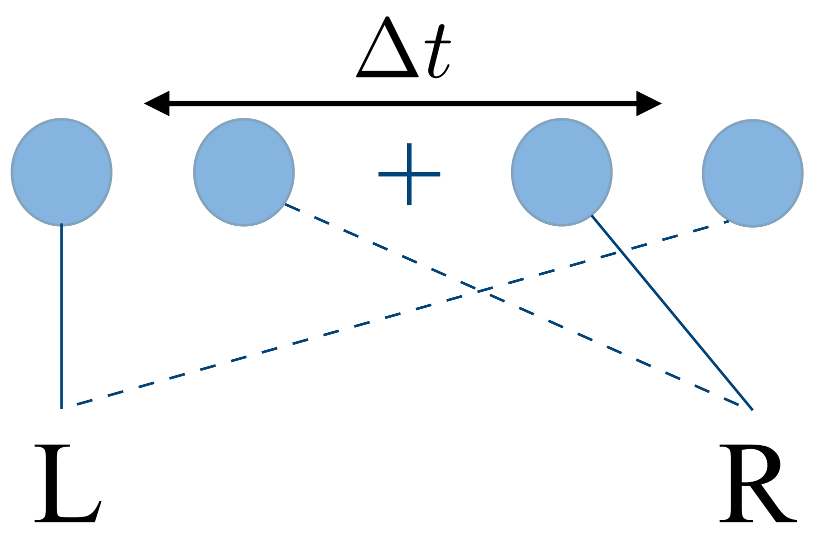
Imaginary part

A time-bin entangled electron pair

Quantum superposition of two pairs separated by 30x their width

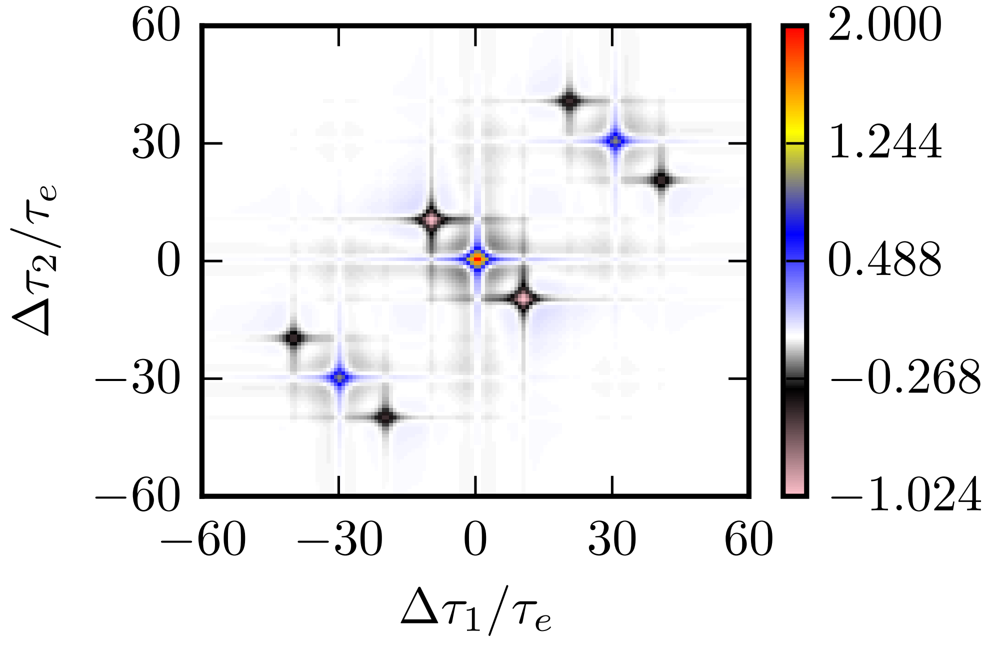


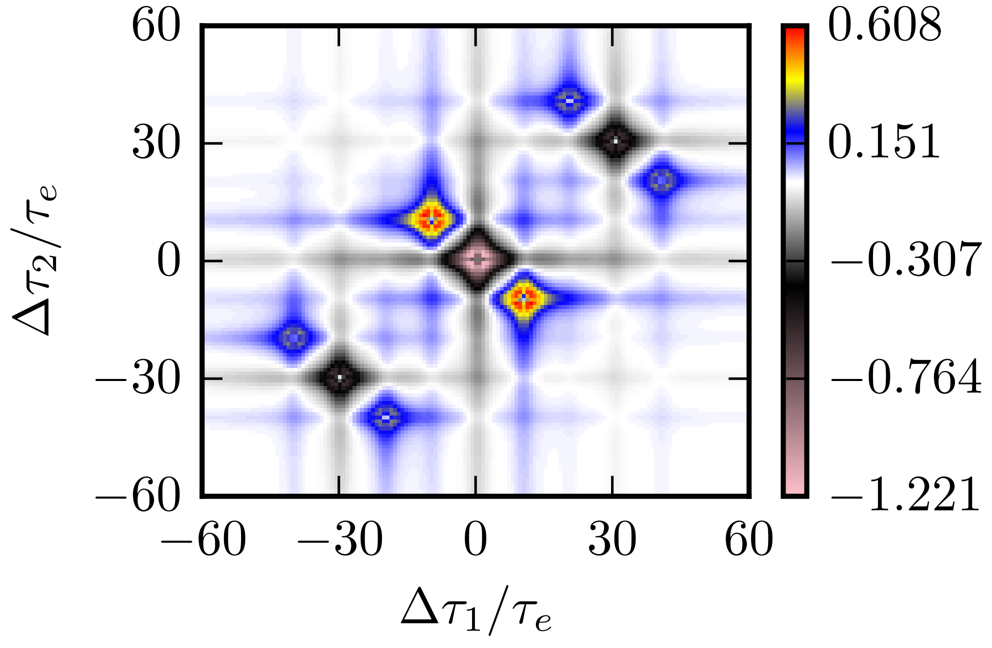
$$\Delta\tau_1 \simeq \Delta\tau_2 \simeq \Delta t$$



$$\Delta\tau_1 \simeq \Delta t + \delta t$$

$$\Delta\tau_2 \simeq \Delta t - \delta t$$





E. Thibierge *et al*, Phys. Rev. B. **93**, 081302(R) (2016)

Completion of the take home message #2

Electron quantum optics as quantum signal processing

Mach-Zehnder interferometry	$\Delta\mathcal{G}^{(e)}$	MZI	$\langle i_{\mathrm{dc}}^{(\mathrm{out})}\rangle$	*linear filtering*
Hong Ou Mandel interferometry	$\Delta\mathcal{G}_1^{(e)}$, $\Delta\mathcal{G}_2^{(e)}$	HOM	$S_i^{(\mathrm{out})}(\omega=0)$	*overlap*
Franson interferometry	$\Delta\mathcal{G}^{(2e)}$	Fr.I	$\Delta S_{12}^{(\mathrm{out})}$	*linear filtering*

Perspectives

From electronic coherences to quantum information quantities: quantitative criteria for *2e* entanglement ?

Coulomb interaction effects on two-electron coherence

E. Thibierge *et al*, Phys. Rev. B **93**, 081302 (2016)
B. Roussel *et al*, Physica Status Solidi B **254**, 1600621 (2017)