

# DPPs & Fermions

Lille 2019, February 6–8

## Titles and Abstracts

**Luis Daniel ABREU.**

Title: *Weyl-Heisenberg ensembles: hyperuniform DPPs related to the Integer Quantum-Hall Effect.*

Abstract: I plan to first present the (infinite) Weyl-Heisenberg ensemble (WHE), a DPP in  $\mathbb{R}^{2d}$  with correlations given by the reproducing kernel of the image of a Short-Time Fourier Transform, depending on a function  $g$ . When  $g$  is a gaussian function, we obtain the Ginibre ensemble. For higher order Hermite functions, the DPP has correlations given by the reproducing kernel of higher Landau levels. The infinite WHE is a hyperuniform DPP according to the Torquato-Stillinger definition (fluctuations suppressed at large scales, measured by the growth rate of the number variance).

Then I will talk about the finite WHE. The finite WHE is constructed as a smooth restriction of the infinite WHE to bounded regions of  $\mathbb{R}^{2d}$ , leading to geometric flexible droplets. The construction boils down to the finite Ginibre ensemble in the case of Gaussian windows and leads to variations of the circular law and measures of the  $L^1$ -deviations, which also indicate suppression of fluctuations at large scales.

The construction is motivated by the Quantum-Hall effect models and suggests a way of modelling the confinement of electrons in higher Landau levels to geometric flexible regions, leading to an extension of the Coulomb gas model which can be lifted to higher Landau levels. The physical interpretation and the trial wave functions resulting from this model will be left for discussion and criticism from the participants.

If time permits and if this is within the scope of the meeting, I can talk also about the relation between DPPs and multitapering methods.

**J r mie BOUTTIER.**

Title: *Fermions in combinatorics: random permutations and partitions.*

Abstract: A classical problem, often attributed to Ulam and Hammersley, is to determine the asymptotic distribution of the maximal length of an increasing subsequence in a random permutation of  $\{1, \dots, n\}$ , as  $n$  gets large. It was fully solved in 1998 by Baik, Deift and Johansson who proved that the rescaled fluctuations are governed by the Tracy-Widom  $\beta = 2$  distribution (which also describes the fluctuations of the largest eigenvalue in the Gaussian Unitary Ensemble of random matrix theory).

In this talk I will present an alternate proof of the BDJ theorem, which is basically due to Okounkov and relies on fermionic techniques. Time allowing I may also discuss some related topics: plane partitions and Schur processes, the boson-fermion correspondence, cylindric partitions and the finite-temperature Airy process, etc.

**Pascal DEGIOVANNI.**

Title: *TBA.*

Abstract: TBA.

**Douglas LUNDHOLM.**

Title: *The art of being a fermion in a sea of many possibilities.*

Abstract: I will try to summarize:

- why fermions appear at all and what are the other possibilities (representations, braids, anyons),
- something about Thomas-Fermi and other density functionals (one-body approximations of the many-body theory),
- the types of wave functions appearing in FQHE (2D) and why we should care about them (Landau levels, Laughlin, quasi-hole and clustered states, parafermions),
- finally suggesting some directions for RMT/Coulomb gas communities.

**Odile MACCHI.**

Title: *Permanents and determinants for bosons and fermions*

Abstract: It is in the context of the Centre scientifique d'Orsay, between 1967 and 1972, and in collaboration with physicists Bernard Picinbono and Christine Benard, that the first general multidimensional probabilistic model for a repulsive point

process was proposed. The idea was to model the detection of a low intensity beam of photons or fermions in thermal equilibrium. The fundamental tool that I used was the formalization of coincidence probabilities (or correlation intensities), which are physically measurable and local quantities. In this talk, I will present:

- conditions for a system of coincidence probabilities to describe a point process;
- the detection process of bosons (in particular, photons) in thermal equilibrium, which is an attractive point process. It is actually a Cox process, i.e., a Poisson process with a random intensity. This intensity is the squared modulus of a Gaussian process, which represents the complex electromagnetic field. The coincidence probabilities of this Cox process can be expressed using permanents, built on the resolvent of a Fredholm equation for the fields covariance;
- the detection process of fermions (in particular, electrons) in a chaotic state, which is a repulsive point process. In the quantum formalism, the system superimposes random wavepackets associated to each fermion. Coincidences can be expressed using Fredholm determinants built on the the resolvent of another Fredholm equation for the packets covariance;
- the physical and statistical symmetry between these two types of particles.

**Gregory SCHEHR.**

Title: *Non-interacting trapped fermions: from random matrices to stochastic growth models.*

Abstract: I was planning to focus my talk on:

- the connection between non-interacting free fermions (in 1d and 2d) in confining traps at zero temperature and random matrix theory (and mention briefly what you can do with that). We actually wrote recently a short review on that topic <https://arxiv.org/abs/1810.12583>.  
I would also like to discuss trapped fermions in higher dimensions  $d > 2$ , where there is obviously no connection to RMT but nevertheless interesting and universal edge fluctuations (described by non-trivial generalizations of the well known Airy kernel).
- discussing the finite temperature extensions of these results, in particular the grand-canonical determinantal process and its connection (in 1d) to the Kardar-Parisi-Zhang (KPZ) universality class and the construction of the so-called "Periodic Airy Process" which describes the time-dependent edge correlations of fermionic systems. If time permits, I would elaborate a bit on the similarities and differences between canonical and grand-canonical ensembles when global fluctuations are considered.
- then I could indeed discuss various topics which have been (up to now) a bit less studied like the effects of interactions and the connection to Calogero-Sutherland models or the problem of non-equilibrium dynamics (the so-called quantum quenches).

**Chris WESTBROOK.**

Title: *The Hanbury Brown Twiss effect for Bosons and Fermions.*

Abstract: TBA.

**Hans ZESSIN.**

Title: *Fascinating facets of Ginibre's Bose gas.*

Abstract: The idea is to arrive to permanent and determinantal processes via Ginibre's Gas, which Ginibre directly derived from quantum mechanical considerations.

- I start with a short informal introduction to Ginibre's work;
- present then shortly Ginibre's Bose gas;
- next I present so-called quantum-classical processes, which interact by a classical pair potential as well as a quantum interaction;
- then, switching off the classical potential, I obtain so-called quantum processes, which are locally specified by an immanant, i.e. a permanent or a determinant;
- these are then shown to be permanent or determinantal processes respectively, which is the topic of the meeting